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# **Glocal Control** A Realization of Global Functions by Local Measurement and Control

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# Why "Glocal Control" ?

Recently, systems to be treated in various fields of engineering including control have became large and complex, and more high level control such as adaptation against changes of environments for open systems is required. Typical example includes meteorological phenomena and bio systems, where our available actions of measurement and control are restricted locally although our main purpose is to achieve the desired global behaviors.

This motivates us to develop a **new research area** so called "Glocal Control," which means that the desired global behaviors is achieved by only local actions.

#### Advanced Science & Technology Driven by Control



## Idea of "Glocal Control"?



# **Urban Heat Island Problem**



## Hierarchical Bio-Network Systems



# Framework for Glocal Control



#### Three Key Issues

1 Paradigm Shift in Control

- Realization of High Quality Products
- → Solving Social Problems



#### **2** New Unified Framework

LTI System with Generalized Frequency Variable
→ Hierarchical Dynamical System with Multiple Resolutions

#### **③ New Control Theory for Systematic Ways of Analysis and Synthesis**

Fundamental Results & New Notions/Principles

→ Practical Applications

#### Three Key Issues

**Idea of Paradigm Shift in Control** Glocal Control **Realization of High Quality Products** → Solving Social Problems Toward Glocal **(2)** New Unified Framework **Control** LTI System with Generalized Frequency Variable → Hierarchical Dynamical System with Multiple Resolutions **(3)** New Control Theory for Systematic Ways

of Analysis and Synthesis *Fundamental Results* & New Notions/Principles → Practical Applications

# OUTLINE

#### **1. Glocal Control**

- 2. Unified Framework with Stability Conditions
- 3. Cooperative Stabilization
- 4. Robust Stability Analysis
- 5. Hierarchical Consensus
- 6. Conclusion

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#### **1. Glocal Control**

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(Hara et al.: CDC2007, Tanaka et al.: ASCC2009)

#### LTI System with Generalized Frequency Variable

A unified representation for multi-agent dynamical systems



Group Robot

Gene Reg. Networks

#### An Example : Cyclic Pursuit



 $\delta \theta_i(t) \rightarrow 2\pi/9$ 

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#### Stability Region for LTISwGFV

(Hara et al. IEEE CDC, 2007)

♦ <u>Define</u>: Domains  $\Omega_+ := \phi(\mathbb{C}_+), \quad \Omega_+^c := \mathbb{C} \setminus \Omega_+$ 



How to characterize the region ? How to check the condition ?

#### Stability Tests for LTISwGFV

Graphical	Algebraic	Numeric (LMI)
Nyquist – type	Hurwitz – type	Lyapunov – type
Polyak & Tsypkin (1996) Fax & Murray (2004) Hara et al. (2007)	Tanaka, Hara, Iwasaki (ASCC2009)	Tanaka, Hara, Iwasaki (ASCC2009)
$h(s)$ and $\sigma(A)$	$h(s)$ and $\sigma(A)$	h(s) and $A$

Hurwitz test for complex coefficients Generalized Lyapunov Inequality<sup>15</sup>

#### **Stability Conditions**

(Tanaka et al., ASCC, 2009)

Given 
$$h(s) = n(s)/d(s)$$
,  $A$   $\mathcal{H}_{A}(s)$  is stable  
 $\sigma(A) \subset \Lambda := \{ \lambda \in \mathbb{C} \mid d(s) - \lambda n(s) \text{ is Hurwitz stable } \}$   
Hermina  
Algebraic condition  
 $\sigma(A) \subset \bigcap_{k=1}^{r} \Sigma_{k}$   
 $\Sigma_{k} := \{ \lambda \in \mathbb{C} \mid l_{k}(\lambda)^{*} \Phi_{k} l_{k}(\lambda) > 0 \}$   
 $(k = 1, 2, ..., \nu)$   
Generalized Lyapunov inequality  
 $X_{k} = X_{k}^{T} > 0 \text{ s.t. } L_{k}(A)^{T}(\Phi_{k} \otimes X_{k})L_{k}(A) > 0$   
for each  $k = 1, 2, ..., \nu$   
 $h(k) = 1, ...$ 

#### Numerical Example: 2<sup>nd</sup> order (1/2)

Given 
$$h(s) = \frac{2s+1}{s^2+s+1}, A \in \mathbb{R}^{n \times n}$$

 $\sigma(A) \subset \Lambda := \{ \ \lambda \in \mathbb{C} \ | \ (s^2 + s + 1) - \lambda(2s + 1) \text{ is Hurwitz stable } \}$ 

Extended Routh-Hurwitz Criterion





# Numerical Example : 2<sup>nd</sup> order (2/2)

Given 
$$h(s) = \frac{2s+1}{s^2+s+1}, A \in \mathbb{R}^{n \times n}$$

$$\sigma(A) \subset \Sigma := \left\{ \begin{array}{l} \lambda \in \mathbb{C} \mid \Delta_1 > 0 \text{ and } \Delta_2 > 0 \end{array} \right\}$$
$$\Delta_1 = \begin{bmatrix} 1 \\ \lambda \end{bmatrix}^* \begin{bmatrix} 1 & -1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ \lambda \end{bmatrix} > 0$$
$$\Delta_2 = \frac{1}{4} \begin{bmatrix} 1 \\ \lambda \\ \lambda^2 \end{bmatrix}^* \begin{bmatrix} 4 & -10 & 7 \\ -10 & 18 & -8 \\ 7 & -8 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ \lambda \\ \lambda^2 \end{bmatrix} > 0$$



Generalized Lyapunov inequality

$$X_{1} = X_{1}^{T} > 0 \quad \text{s.t.} \quad \begin{bmatrix} I \\ A \end{bmatrix}^{T} \left( \begin{bmatrix} 1 & -1 \\ -1 & 0 \end{bmatrix} \otimes X_{1} \right) \begin{bmatrix} I \\ A \end{bmatrix} > 0$$
$$X_{2} = X_{2}^{T} > 0 \quad \text{s.t.} \quad \begin{bmatrix} I \\ A \\ A^{2} \end{bmatrix}^{T} \left( \begin{bmatrix} 4 & -10 & 7 \\ -10 & 18 & -8 \\ 7 & -8 & 0 \end{bmatrix} \otimes X_{2} \right) \begin{bmatrix} I \\ A \\ A^{2} \end{bmatrix} > 0$$

#### Algorithm



#### Numerical Example: 4th order

$$h(s) = \frac{100(s+2)(\frac{19}{10}s^2 - \frac{1}{500000}s + \frac{21}{10})}{(s-1)^2(s+1)(s+100)}$$

#### Unstable & NMP



#### An Application : Biological rhythms

#### **Motivation**

Biological rhythms



- 24h-cycle, heart beat, sleep cycle etc.
- caused by periodic oscillations of protein concentrations in <u>Gene Regulatory Networks</u>

#### Medical and engineering applications

- Artificially engineered biological oscillators (e.g.) Repressilator [Elowitz & Leibler, *Nature*, 2000]





What are analytic conditions for convergence and the existence of oscillations ?

#### Condition for Existence of Oscillations

The cyclic GRN has periodic oscillations if at least one of eigenvalue of K lies inside  $\Omega_+$ where  $\Omega_{+} := \left\{ \lambda \in \mathbb{C} \mid \exists s \in \mathbb{C}_{+} \text{ s.t. } \lambda = \phi(s) \right\}$ 



 $\phi(s) := (T_a s + 1)(T_b s + 1)$ 

#### Analytic Criteria

#### • Assumptions: All interactions are repressive

**Theorem** [Hori et al., CDC2009]

The cyclic GRN has periodic oscillations, if an analytic

condition in terms of  $(N, \nu, R, Q)$  is satisfied.

 $R := \frac{\sqrt{c\beta}}{\sqrt{ab}}$  Raio of production and degradation rates  $Q := \frac{\sqrt{T_a T_b}}{(T_a + T_b)/2}$  Gap between two tme constants



$$T\simeq rac{2\pi Q an(rac{\pi}{N})}{\sqrt{1+Q^2 an^2(rac{\pi}{N})}-1}\sqrt{T_aT_b}$$

(Hori, Hara: CDC2010)

#### Message : Framework and Stability

1 LTI system with generalized freq. variable a proper class of homogeneous multi-agent dynamical systems

② Three types of stability tests, namely graphical, algebraic, and numeric (LMI) powerful tools for analysis

③ Parametric stability analysis for gene regulatory networks new biological insight

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(Hara et al.: CDC-CCC2009)

## An Application: Inverted Pendulum

Cooperatively stabilizable ?



<u>**Remarks</u>** : No physical interactions memory-less feedback</u>







```
\Omega^c_+ is non-empty.
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**Solely stabilizable**:

$$\Omega^c_+$$
 interacts the real axis.

*N* : odd : Coop. Stab. = Solely Stab.

N: even : Coop Stab. (N=2)  $\rightarrow$  any N=2m

## Property 2

$$\mathcal{H}_{A}(s) := \left(\frac{d(s)}{n(s)}I - A\right)^{-1} \text{ is stable.} ; h(s) = \frac{n(s)}{d(s)}$$

$$\widehat{1} \quad p(\lambda, s) := d(s) - \lambda n(s)$$
(ii)  $\sigma(A) \subset \Lambda := \left\{ \lambda \in \mathbb{C} \mid p(\lambda, s) \\ \text{ is Hurwitz stable.} \right\}$ 
Solely Stabilizable :  $\bigstar$ 
Stabilizable by a real gain output feedback
Cooperatively Stablizable :  $\bigstar$ 
Stabilizable by a complex gain output feedback
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#### Theorem: Coop. Stabliz. = Soley Stabiliz.

$$h_2(s) = \frac{cs+d}{s^2+as+b}$$

#### **Higher order systems:**

$$\begin{aligned} \mathcal{H}_{0}(s) &\triangleq \{ h(s) = \frac{k}{d(s)} \mid k \neq 0 \} \\ \mathcal{H}_{1}(s) &\triangleq \{ h(s) = \frac{ks}{d(s)} \mid k \neq 0, \ d(0) \neq 0 \} \\ \mathcal{H}_{2}(s) &\triangleq \{ h(s) = \frac{k(s^{2} - b^{2})}{d(s)} \mid k \neq 0, \ d(\pm b) \neq 0 \} \\ d(s) &= s^{n} + a_{n-1}s^{n-1} + \dots + a_{1}s + a_{0} \end{aligned}$$

#### Example: Inverted Pendulum

$$P_{\theta}(s) = rac{-m\ell s}{D(s)} \in \mathcal{H}_1(s)$$

$$D(s) = a_3 s^3 + a_2 s^2 + a_1 s + a_0 ,$$

$$a_{3} := \frac{1}{3}(4M+m)m\ell^{2},$$

$$a_{2} := (M+m)\mu_{p} + \frac{4}{3}\mu_{t}m\ell^{2},$$

$$a_{1} := -(M+m)mg\ell + \mu_{p}\mu_{t},$$

$$a_{0} := -\mu_{t}mg\ell.$$



#### An example : Cope. Stab. $\pm$ Soley Stab.



#### Inverted Pendulum : PD control (1/2)



$$h(s) = \frac{(Ts+1)(\frac{19}{10}s^2 - \frac{1}{500}s + \frac{21}{10})}{s(s-2)(s+1)(s+5)}$$

We can prove by a symbolic computation (QE) that the system can not be stabilized alone no matter how we choose T>0.



#### Inverted Pendulum : PD control (2/2)

$$T = 1/2: \quad h(s) = \frac{(\frac{1}{2}s+1)(\frac{19}{10}s^2 - \frac{1}{500}s + \frac{21}{10})}{s(s-2)(s+1)(s+5)}$$



#### Message : Cooperative Stabilization

**1** Cooperation realizes complex gain feedback virtually.

**(2)** Different roles of two agents are required for getting an advantage for stabilization.

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(Hara et al.: CDC2010)

Robust Stability for LTI Systems with GFV

# **Fundamental Questions in Control**

From <u>Stability</u> to

- <u>Robust Stability</u> ?
   \* homogeneous -> heterogeneous
   \* physical inter-agent interactions
- <u>Control Performance</u> ?

H∞-norm Computation ?



## Robust Stability Condition for Heterogeneous Perturbations

(Hara et al.: CDC2010)

#### <u>Assumption</u>

 $\exists D$ : diagonal s.t.  $DAD^{-1}$  is normal

**<u>Theorem</u>**: The following conditions are equivalent. (i) The system is robustly stable for  $\Delta_{d\gamma}$ . (ii)  $\left\|\frac{\lambda h}{1-\lambda h}\right\|_{\infty} < \gamma, \quad \forall \ \lambda \in \sigma(A)$ (iii)  $\left| \frac{\lambda}{\phi - \lambda} \right| < \gamma, \ \forall \ \lambda \in \sigma(A),$  $\forall \ \phi \in \Phi := \{1/h(j\omega) | \ \omega \in \mathbb{R} \}.$ 

#### Linearized Gene Network Model



#### **Robust Stability Test**



#### The smaller values of N, Q, and/or R $\rightarrow$ more robust for maintaining stability

(Osawa et. al., ASCC2011) tomorrow morning

#### Stability for Dissipative Agents

<u>Agent Dynamics</u> — SISO (Q, S, R)-dissipative

$$\dot{x}_i = f_i(x_i) + g_i(x_i)u_i$$
$$y_i = h_i(x_i)$$

$$Q = \text{diag}\{Q_i\} \le 0,$$
  

$$S = \text{diag}\{S_i\},$$
  

$$R = \text{diag}\{R_i\} > 0.$$

## Theorem (LMI)

If  $\exists$  a diagonal matrix D > 0 such that

$$A^T \mathbf{D} R A + \mathbf{D} S A + A^T S^T \mathbf{D} + \mathbf{D} Q < \mathbf{0}$$

holds, then the network of N interconnected  $(Q_i, S_i, R_i)$ -dissipative agents is asymptotically stable.

#### Message : Robust Stability

(1) Methods of robust stability analysis for standard systems such as *D*-scaling work well for stability analysis for heterogeneous multiagent dynamical systems.

② Although the results are not complete, there are many potential practical application fields to which we can apply them.

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(Shimizu, Hara: SICE2008, Hara et al.: ACC2009)

#### Hierarchical Consensus Problem

$$\dot{x}(t) = Ax(t)$$
  $\exists \xi, \lim_{t \to \infty} x(t) = \xi \cdot 1$ 



# total agents :  $n1 \times n2 \times n3$ 

#### **Hierarchical Structure**



#### **Eigenvalue Distributions**



#### Time Responses (n1=25, n2=4)



#### Message : Hierarchical Consensus

**1 Proper ways of aggregation and distribution are important to achieve rapid consensus.** 

**(2)** Low rankness of interlayer connection captures them properly.

# **Toward "Glocal Control"**

A Unified Framework for

**Decentralized Cooperative Control** 

of Large Scale Networked Dynamical Systems

Key Idea : Dynamical System with Generalized Frequency Variable

Stability & Robust Stability Analysis Cooperative Stabilization

<u>Extensions</u> :

- Hierarchical case
- Non-linear case
- Control Performances, Synthesis ?

#### New Framework for System Theory





#### Image of Glocal Control System



## Smart Energy NW and Energy Saving

#### **Smart Energy Network**

Electric power network + Gas energy network



http://tinycomb.com/wp-content/ uploads/2009/05/smart-grid.jpg



Hierarchical Air Conditioning System Group of buildings Set of floors Set of rooms

#### **Evacuation Guidance for Tsunami**



How to set up GPS wave sensors to predict the time and height of "tsunami" properly for effective evacuation guidance ? *Optimal time-, space-, level- resolution* ?

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Please join us to develop "Glocal Control Theory" and to solve social problems through "Glocal Control"

