Glocal Control
A Realization of Global Functions by Local Measurement and Control

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Recently, systems to be treated in various fields of engineering including control have became large and complex, and more high level control such as adaptation against changes of environments for open systems is required. Typical example includes meteorological phenomena and bio systems, where our available actions of measurement and control are restricted locally although our main purpose is to achieve the desired global behaviors.

This motivates us to develop a new research area, so called "Glocal Control," which means that the desired global behaviors is achieved by only local actions.
Advanced Science & Technology Driven by Control

Glocal Control

Realization of High Quality Products → Solving Social Problems such as Energy, Environments, and Medicine

Hybrid Control

Multiple Functions

High Performance

Robust Control

Modern Control

Classical Control

Stabilization

Automation

Realization of High Quality Products

Solving Social Problems such as Energy, Environments, and Medicine

Steel process

Linear motor car

Engine control

Robotics

Meteorological Phenomena

Bio-system

Power NWs

Transportation

Realization of High Quality Products → Solving Social Problems such as Energy, Environments, and Medicine
Idea of "Glocal Control"?

Real World

Locally

Control

Large-scale & Complex

Global Behaviors By Locally

Prediction

Locally

Measurement

Meteorological Phenomena
Power NWs
Transportation NWs
Bio-system

Locally
Urban Heat Island Problem

Control

Prediction

Measurement

Glocal Control

Fans

Temperature, Wind Sensors

Where (space resolution)
When (time resolution)
How (level resolution)
Hierarchical Bio-Network Systems

Layer 1: GENE Regulatory Networks

Layer 2: CELL Signaling Networks

Layer 3: METABOLIC Networks

Cyclic Structure:

- Subsystem 1
- Subsystem 2
- Subsystem 3
- Subsystem 4
Framework for Glocal Control

Real World

Physical Network

Locally

Dual Network

(1) Hierarchical Dynamical Systems with Multi-resolution

(Dynamical Logical Network (Measurement, Prediction))

Power NWs

Transportation NWs

Meteorological Phenomena

Bio-system

Control Locally

Measurement Locally

Real World
Three Key Issues

① Paradigm Shift in Control
  Realization of High Quality Products
  → Solving Social Problems

② New Unified Framework
  LTI System with Generalized Frequency Variable
  → Hierarchical Dynamical System with Multiple Resolutions

③ New Control Theory for Systematic Ways of Analysis and Synthesis
  Fundamental Results & New Notions/Principles
  → Practical Applications
Three Key Issues

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   Realization of High Quality Products
   ➔ Solving Social Problems

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   \emph{LTI System with Generalized Frequency Variable}
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   \emph{Fundamental Results} & New Notions/Principles
   ➔ Practical Applications
1. Glocal Control
2. Unified Framework with Stability Conditions
3. Cooperative Stabilization
4. Robust Stability Analysis
5. Hierarchical Consensus
6. Conclusion
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(Hara et al.: CDC2007, Tanaka et al.: ASCC2009)
LTI System with Generalized Frequency Variable

A unified representation for multi-agent dynamical systems

\[ C(sI - A)^{-1} B + D \]

\[ \frac{1}{s} \Rightarrow h(s) \]

\[ \Phi(s) = \frac{1}{h(s)} \]

Group Robot

Gene Reg. Networks

Dynamics + Information Structure
An Example: Cyclic Pursuit

\[ \delta \theta_i(t) \rightarrow \frac{2\pi}{9} \]
Define: Domains \( \Omega_+ := \phi(C_+) \), \( \Omega^c_+ := \mathbb{C} \setminus \Omega_+ \)

All eigenvalues belong to…

How to characterize the region? How to check the condition?
## Stability Tests for LTISwGFV

<table>
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<th>Graphical</th>
<th>Algebraic</th>
<th>Numeric (LMI)</th>
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<td>Nyquist – type</td>
<td>Hurwitz – type</td>
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<tr>
<td>$h(s)$ and $\sigma(A)$</td>
<td>$h(s)$ and $\sigma(A)$</td>
<td>$h(s)$ and $A$</td>
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- **Hurwitz test for complex coefficients**
- **Generalized Lyapunov Inequality**
Stability Conditions

Given \( h(s) = n(s)/d(s), \quad A \)  
\( \mathcal{H}_A(s) \) is stable

\[ \sigma(A) \subseteq \Lambda := \{ \lambda \in \mathbb{C} \mid d(s) - \lambda n(s) \text{ is Hurwitz stable} \} \]

**Algebraic condition**

\[ \sigma(A) \subseteq \bigcap_{k=1}^\nu \Sigma_k \]

\[ \Sigma_k := \{ \lambda \in \mathbb{C} \mid l_k(\lambda)^* \Phi_k l_k(\lambda) > 0 \} \]

\( (k = 1, 2, \ldots, \nu) \)

**Extended Routh-Hurwitz Criterion** [Frank, 1946]

**Generalized Lyapunov inequality**

**LMI feasibility problem**

\[ X_k = X_k^T > 0 \text{ s.t. } L_k(A)^T (\Phi_k \otimes X_k) L_k(A) > 0 \]

for each \( k = 1, 2, \ldots, \nu \)

\[ l_\nu(\lambda) := \begin{bmatrix} 1 \\ \lambda \\ \vdots \\ \lambda^{\nu-1} \end{bmatrix}, \quad L_\nu(A) := \begin{bmatrix} I \\ A \\ \vdots \\ A^{\nu-1} \end{bmatrix} \]
Given \( h(s) = \frac{2s + 1}{s^2 + s + 1}, \ A \in \mathbb{R}^{n \times n} \)

\[ \sigma(A) \subset \Lambda := \{ \lambda \in \mathbb{C} \mid (s^2 + s + 1) - \lambda(2s + 1) \text{ is Hurwitz stable} \} \]

Extended Routh-Hurwitz Criterion

\[ \sigma(A) \subset \Sigma := \{ \lambda \in \mathbb{C} \mid \Delta_1 > 0 \text{ and } \Delta_2 > 0 \} \]

\[ \Delta_1 = \left[ \frac{1}{\lambda} \right]^* \begin{bmatrix} 1 & -1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ \lambda \end{bmatrix} > 0 \]

\[ \Delta_2 = \frac{1}{4} \left[ \frac{1}{\lambda^2} \right]^* \begin{bmatrix} 4 & -10 & 7 \\ -10 & 18 & -8 \\ 7 & -8 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ \lambda \\ \lambda^2 \end{bmatrix} > 0 \]
Numerical Example: 2nd order (2/2)

Given \( h(s) = \frac{2s + 1}{s^2 + s + 1} \), \( A \in \mathbb{R}^{n \times n} \)

\( \sigma(A) \subset \Sigma := \{ \lambda \in \mathbb{C} \mid \Delta_1 > 0 \text{ and } \Delta_2 > 0 \} \)

\[
\Delta_1 = \begin{bmatrix} 1 \\ \lambda \end{bmatrix}^* \begin{bmatrix} 1 & -1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ \lambda \end{bmatrix} > 0
\]

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\Delta_2 = \frac{1}{4} \begin{bmatrix} 1 \\ \lambda \\ \lambda^2 \end{bmatrix}^* \begin{bmatrix} 4 & -10 & 7 \\ -10 & 18 & -8 \\ 7 & -8 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ \lambda \\ \lambda^2 \end{bmatrix} > 0
\]

Generalized Lyapunov inequality

\[
X_1 = X_1^T > 0 \quad \text{s.t.} \quad \begin{bmatrix} I \\ A \\ A^2 \end{bmatrix}^T \left( \begin{bmatrix} 1 & -1 \\ -1 & 0 \end{bmatrix} \otimes X_1 \right) \begin{bmatrix} I \\ A \\ A^2 \end{bmatrix} > 0
\]

\[
X_2 = X_2^T > 0 \quad \text{s.t.} \quad \begin{bmatrix} I \\ A \\ A^2 \end{bmatrix}^T \left( \begin{bmatrix} 4 & -10 & 7 \\ -10 & 18 & -8 \\ 7 & -8 & 0 \end{bmatrix} \otimes X_2 \right) \begin{bmatrix} I \\ A \\ A^2 \end{bmatrix} > 0
\]
Algorithm $h2Phi(h(s))$:

**Input:** $h(s) = \frac{b_1 s^{\nu-1} + \cdots + b_\nu}{s^\nu + a_1 s^{\nu-1} + \cdots + a_\nu}$

**Output:** $\ell_k$ and $\Phi_k$

1. $p_0 \leftarrow 1$, $q_0 \leftarrow 0$
   
   for $i \leftarrow 1$ until $2\nu - 1$
   
   if $i \leq \nu$
   
   $p_i \leftarrow a_i - b_i x$, $q_i \leftarrow -b_i y$
   
   else
   
   $p_i \leftarrow 0$, $q_i \leftarrow 0$

2. $\Delta_1 \leftarrow p_1$
   
   for $k \leftarrow 2$ until $2\nu$
   
   $M \leftarrow O((2k-1) \times (2k-1))$
   
   $\Delta_k(\lambda + \bar{\lambda})/2, (\lambda - \bar{\lambda})/2j$

3. $\bar{\lambda} \leftarrow \text{minimum of the degree of } \lambda \text{ in } \Delta_k^*(\lambda, \bar{\lambda}) - 1$

4. $\bar{\lambda} \leftarrow 0$

   for $k \leftarrow 2$ until $\ell_k - 1$

   $\Phi_k(m + 1, l + 1)$

   $\leftarrow \text{the coefficient of } \lambda^m \bar{\lambda}^l \text{ in } \Delta_k^*(\lambda, \bar{\lambda})$

5. return $\ell_k$ and $\Phi_k$.

---

**Given Data:** all coefficients of numerator and denominator of $h(s)$

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**Algorithm**

---

**Systematic methods for stability analysis**

**Hurwitz-type & LMI**

---

**Result:**

$\Phi_k(s)$ $(k = 1, 2, \cdots, \nu)$
Numerical Example: 4th order

\[ h(s) = \frac{100(s + 2)(\frac{19}{10}s^2 - \frac{1}{500000}s + \frac{21}{10})}{(s - 1)^2(s + 1)(s + 100)} \]

\[ \Delta_1 > 0 \]

\[ \Delta_2 > 0 \]

\[ \Delta_3 > 0 \]

\[ \Delta_4 > 0 \]

Stability Region

Unstable & NMP
An Application: Biological rhythms

Motivation

• Biological rhythms
  – 24h-cycle, heart beat, sleep cycle etc.
  – caused by periodic oscillations of protein concentrations in **Gene Regulatory Networks**

• Medical and engineering applications
  – Artificially engineered biological oscillators
    (e.g.) Repressilator [Elowitz & Leibler, *Nature*, 2000]
What are analytic conditions for convergence and the existence of oscillations?
The cyclic GRN has periodic oscillations if at least one of eigenvalue of $K$ lies inside $\Omega_+$ where

$$\Omega_+ := \left\{ \lambda \in \mathbb{C} \mid \exists s \in \mathbb{C}_+ \text{ s.t. } \lambda = \phi(s) \right\}$$

$$\phi(s) := (T_\alpha s + 1)(T_\beta s + 1)$$
• **Assumptions:** All interactions are repressive

**Theorem** [Hori et al., CDC2009]

The cyclic GRN has periodic oscillations, if an analytic condition in terms of \((N, \nu, R, Q)\) is satisfied.

\[
R := \frac{\sqrt{c\beta}}{\sqrt{ab}} \quad \text{Raio of production and degradation rates}
\]

\[
Q := \frac{\sqrt{TaTb}}{(Ta+Tb)/2} \quad \text{Gap between two time constants}
\]

\[
T \approx \frac{2\pi Q \tan(\frac{\pi}{N})}{\sqrt{1 + Q^2 \tan^2(\frac{\pi}{N}) - 1}} \sqrt{TaTb}
\]

(Hori, Hara: CDC2010)
① LTI system with generalized freq. variable
   a proper class of homogeneous multi-agent
dynamical systems

② Three types of stability tests, namely
   graphical, algebraic, and numeric (LMI)
   powerful tools for analysis

③ Parametric stability analysis for gene
   regulatory networks
   new biological insight
OUTLINE

1. Glocal Control
2. Unified Framework with Stability Conditions
3. Cooperative Stabilization
4. Robust Stability Analysis
5. Hierarchical Consensus
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(Hara et al.: CDC-CCC2009)
An Application: Inverted Pendulum

Cooperatively stabilizable?

Not stabilizable!

Remarks: No physical interactions
memory-less feedback
Property 1

(i) $\sigma(A) \subset \Omega^c_+$

Cooperatively stabilizable:
$\Omega^c_+$ is non-empty.

Solely stabilizable:
$\Omega^c_+$ intersects the real axis.


$N$: even : Coop Stab. ($N=2$) $\Rightarrow$ any $N=2m$
Property 2

$\mathcal{H}_A(s) := \left(\frac{d(s)}{n(s)} I - A\right)^{-1}$ is stable. \( h(s) = \frac{n(s)}{d(s)} \)

$p(\lambda, s) := d(s) - \lambda n(s)$

(ii) $\sigma(A) \subseteq \Lambda := \{ \lambda \in \mathbb{C} \mid p(\lambda, s) \text{ is Hurwitz stable.} \}$

Solely Stabilizable :  
Stabilizable by a **real gain** output feedback

Cooperatively Stabilizable :  
Stabilizable by a **complex gain** output feedback
2\textsuperscript{nd} order systems:

\[ h_2(s) = \frac{cs + d}{s^2 + as + b} \]

Higher order systems:

\[ \mathcal{H}_0(s) \triangleq \{ h(s) = \frac{k}{d(s)} \mid k \neq 0 \} \]

\[ \mathcal{H}_1(s) \triangleq \{ h(s) = \frac{ks}{d(s)} \mid k \neq 0, d(0) \neq 0 \} \]

\[ \mathcal{H}_2(s) \triangleq \{ h(s) = \frac{k(s^2 - b^2)}{d(s)} \mid k \neq 0, d(\pm b) \neq 0 \} \]

\[ d(s) = s^n + a_{n-1}s^{n-1} + \cdots + a_1s + a_0 \]
Example: Inverted Pendulum

\[ P_\theta(s) = \frac{-mls}{D(s)} \in \mathcal{H}_1(s) \]

\[ D(s) = a_3 s^3 + a_2 s^2 + a_1 s + a_0, \]

\[ a_3 := \frac{1}{3} (4M + m) ml^2, \]

\[ a_2 := (M + m) \mu_p + \frac{4}{3} \mu_t ml^2, \]

\[ a_1 := -(M + m) mgl + \mu_p \mu_t, \]

\[ a_0 := -\mu_t mgl. \]
An example: Cope. Stab. ≠ Soley Stab.

\[ h(s) = \frac{100(s + 2)(\frac{19}{10}s^2 - \frac{1}{500000} s + \frac{21}{10})}{(s - 1)^2(s + 1)(s + 100)} \]

\[ \lambda = (\frac{-1 \pm j}{2})/2 \]

Follower

Control Law

\[ \begin{align*}
    u_1 &= -(y_1 - y_2)/2 \\
    u_2 &= -(y_1 + y_2)/2
\end{align*} \]

Leader

Stability Region
We can prove by a symbolic computation (QE) that the system cannot be stabilized alone no matter how we choose $T > 0$. 

\[ h(s) = \frac{(Ts + 1)(\frac{19}{10}s^2 - \frac{1}{500}s + \frac{21}{10})}{s(s - 2)(s + 1)(s + 5)} \]
Stability Region

\[ T = \frac{1}{2} : \quad h(s) = \frac{\left( \frac{1}{2} s + 1 \right) \left( \frac{19}{10} s^2 - \frac{1}{500} s + \frac{21}{10} \right)}{s(s - 2)(s + 1)(s + 5)} \]

\[ \lambda = 1.5 \pm 12j \]

\[ A = \begin{bmatrix} 1.5 & -12 \\ 12 & 1.5 \end{bmatrix} \]
Message: Cooperative Stabilization

① Cooperation realizes complex gain feedback virtually.

② Different roles of two agents are required for getting an advantage for stabilization.
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(Hara et al.: CDC2010)
Robust Stability for LTI Systems with GFV

Fundamental Questions in Control

From *Stability* to

- *Robust Stability*?
  - homogeneous $\rightarrow$ heterogeneous
  - physical inter-agent interactions
- *Control Performance*?
  - $H^\infty$-norm Computation?
Multiplicative Perturbations

\[ \tilde{H}(s) = (I + \Delta(s)) \cdot h(s) \]

Nominal system: homogeneous

Independent perturbations

\[ \Delta_{d\gamma} := \{ \Delta(s) | \Delta(s) = \text{diag}\{\delta_i(s)\}, \|\Delta(s)\|_\infty \leq 1/\gamma \} \]
Robust Stability Condition for Heterogeneous Perturbations

(Hara et al.: CDC2010)

**Assumption**
\[ \exists D : \text{diagonal s.t. } DAD^{-1} \text{ is normal} \]

**Theorem:** The following conditions are equivalent.

(i) The system is robustly stable for \( \Delta_d \gamma \).

(ii) \[ \left\| \frac{\lambda h}{1 - \lambda h} \right\|_{\infty} < \gamma, \ \forall \ \lambda \in \sigma(A) \]

(iii) \[ \left| \frac{\lambda}{\phi - \lambda} \right| < \gamma, \ \forall \ \lambda \in \sigma(A), \text{ with } \phi \in \Phi := \left\{ \frac{1}{h(j\omega)} \middle| \ \omega \in \mathbb{R} \right\}. \]
Each gene’s dynamics

\[ h_i(s) = (1 + \delta_i(s))h(s) \]

Interaction structure

\[
\begin{bmatrix}
0 & 0 & 0 & \cdots & R_2^2 f_2'(p_1^*) \\
0 & 0 & 0 & \cdots & 0 \\
0 & R_3^2 f_3'(p_2^*) & 0 & \cdots & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
0 & 0 & 0 & \cdots & R_N^2 f_N'(p_{N-1}^*)
\end{bmatrix}
\]

\[ \exists D: \text{ diagonal s.t. } DAD^{-1} \text{ is normal} \]
∀ λ ∈ σ(A) and ω

\[ \left| \frac{1}{\phi(j\omega) - \lambda} \right| < \frac{\gamma}{|\lambda|} \]

\[ L := R^2 \prod_{\ell=1}^{N} |\kappa_{\ell}|^{\frac{1}{N}} \]

\[ |\phi(j\omega) - \lambda| > \frac{L}{\gamma} \]

The smaller values of \( N, Q, \) and/or \( R \) → more robust for maintaining stability

(Osawa et. al., ASCC2011) tomorrow morning
Agent Dynamics

\[ \begin{align*}
\dot{x}_i &= f_i(x_i) + g_i(x_i)u_i \\
y_i &= h_i(x_i)
\end{align*} \]

SISO \((Q, S, R)\)-dissipative

\[ Q = \operatorname{diag}\{Q_i\} \leq 0, \quad S = \operatorname{diag}\{S_i\}, \quad R = \operatorname{diag}\{R_i\} \geq 0. \]

Theorem (LMI)

If \( \exists \) a diagonal matrix \( D > 0 \) such that

\[ A^T D R A + D S A + A^T S^T D + D Q < 0 \]

holds, then the network of \( N \) interconnected \((Q_i, S_i, R_i)\)-dissipative agents is asymptotically stable.

(Hirsch, Hara: IFAC2008)
① Methods of robust stability analysis for standard systems such as $D$-scaling work well for stability analysis for heterogeneous multi-agent dynamical systems.

② Although the results are not complete, there are many potential practical application fields to which we can apply them.
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(Shimizu, Hara: SICE2008, Hara et al.: ACC2009)
Hierarchical Consensus Problem

\[ \dot{x}(t) = Ax(t) \quad \exists \xi, \lim_{t \to \infty} x(t) = \xi \cdot 1 \]

# total agents: \( n_1 \times n_2 \times n_3 \)
Hierarchical Structure

\[ A_1 = P - I \]

\[ A_l = \text{diag}(A_{l-1} - I) + P \otimes \Delta \]

- Homogeneous structure
- Fractal structure
- Property on Interactions

Low Rank Interaction:
\[ \Delta = 1 \cdot \zeta^T \]

Share an aggregated information
Control uniformly

Weak interaction:
Sparse
Small gain
Eigenvalue Distributions

\[ \triangle: \text{Rank 1} \]

\[
eigs(A_1) = \bigcup_{r=1}^{n_1} \exp\left(2\pi j \frac{(r-1)}{n_1}\right) - 1
\]

\[
eigs(A_2) = \begin{cases} 
\bigcup_{r=1}^{n_2} \exp\left(2\pi j \frac{(r-1)}{n_2}\right) - 1 \\
\bigcup_{r=2}^{n_1} \exp\left(2\pi j \frac{(r-1)}{n_1}\right) - 2
\end{cases}
\]

\[ n_1 = 25 \]

\[ > n_2 = 4 \]

\[ \circ: \text{rank 1} \]

\[ \times: \text{Identity} \]
**Time Responses** (n1=25, n2=4)

Diagram showing the time responses for two different ranks, n1 and n2. The graph illustrates the x-position over time for multiple agents or subsystems. The notation △ represents the difference between n1 and n2, indicating a rapid consensus when n1 > n2. The diagram also highlights the distribution and aggregation processes within the system.
① Proper ways of aggregation and distribution are important to achieve rapid consensus.

② Low rankness of interlayer connection captures them properly.
Toward “Glocal Control”

A Unified Framework for Decentralized Cooperative Control of Large Scale Networked Dynamical Systems

**Key Idea:** Dynamical System with Generalized Frequency Variable

**Stability & Robust Stability Analysis**

**Cooperative Stabilization**

**Extensions:**
- Hierarchical case
- Non-linear case
- Control Performances, Synthesis?
New Framework for System Theory

2D System

$\begin{pmatrix} 1 & 0 \\ 0 & 1/z_2 \end{pmatrix}$

$A \quad B$

$C \quad D$

$y \quad u$

Singular Perturbed System

$\begin{pmatrix} h(s)I & 0 \\ 0 & h(\mu s)I \end{pmatrix}$

$A \quad B$

$C \quad D$

$y \quad u$

Multi-resolved Systems
Image of Glocal Control System

Glocal Adaptor

Inverse model

Generator for local reference commands

Desired Global Behavior

Large scale complex system

Hierarchical Distributed Controller

Local measurement (HR)

Global measurement (LR)

Glocal Predictor

Local Control input
Smart Energy NW and Energy Saving

Smart Energy Network

Electric power network
+ Gas energy network

Hierarchical Air Conditioning System

Group of buildings
Set of floors
Set of rooms

Evacuation Guidance for Tsunami

Wave measurement system by GPS

How to set up GPS wave sensors to predict the time and height of “tsunami” properly for effective evacuation guidance?

Optimal time-, space-, level- resolution?
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Please join us to develop “Glocal Control Theory” and to solve social problems through “Glocal Control”

Thank you very much!