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Robustness in Networked Dynamical Systems

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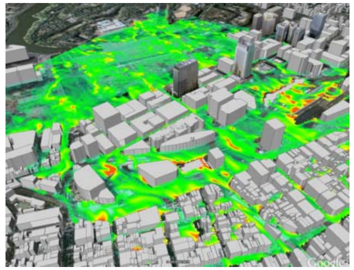
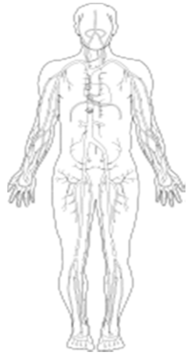
Future Direction in Control

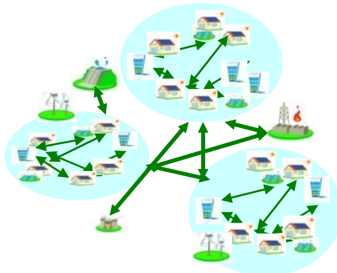
Realization of High Quality Products → Solving Social Problems such as Energy, Environments, and Medicine



Meteorological Phenomena



Bio-systems



Power NWs

Transportation

Multiple Functions — Hybrid Control

High Performance — Robust Control

Automation — Modern Control

Stabilization — Classical Control

Linear motor car

Engine control

Robotics

Mechatronics

Aerospace

Steel process

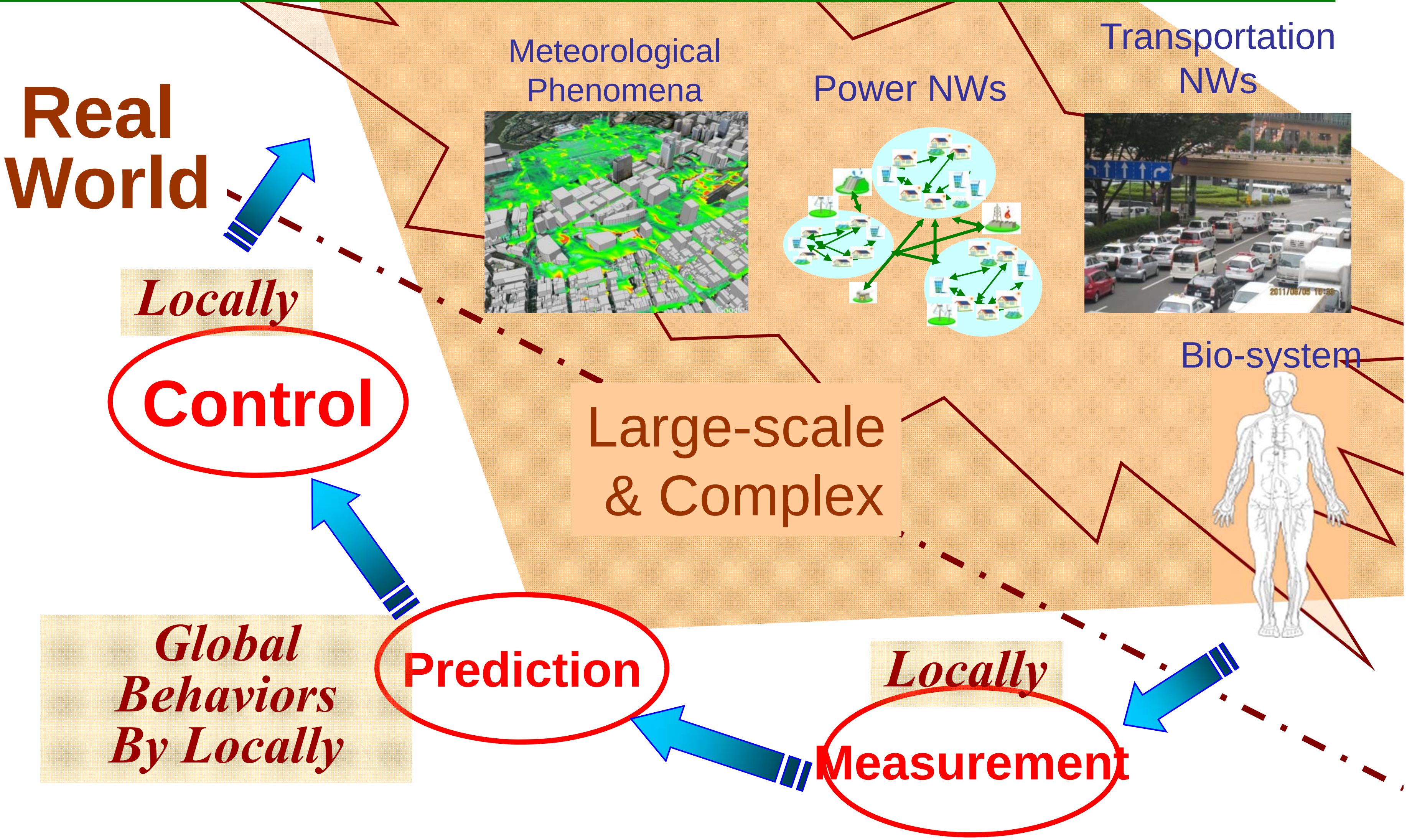
Chemical process

Watt

Glocal Control

Idea of "Glocal Control" ?

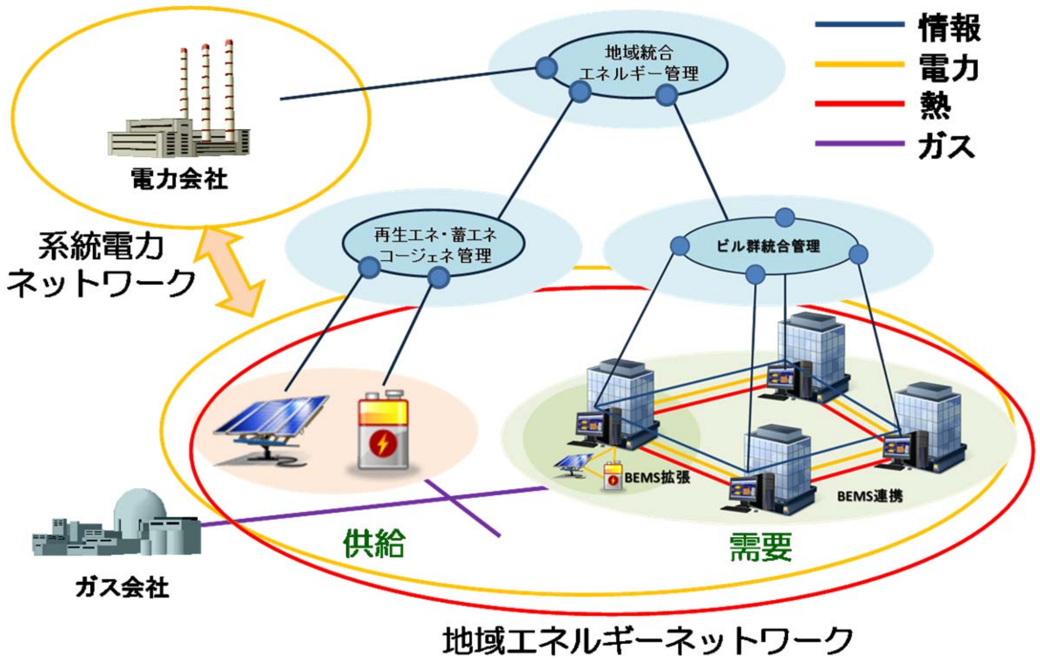
Glocal Control : to achieve **desired global behavior** by only **local actions** of measurement and control



Smart Energy NW and Energy Saving

Smart Energy Network

Electric power network
+ Gas energy network



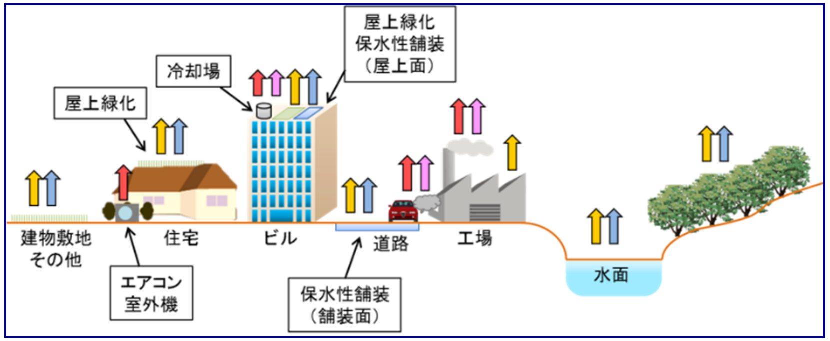
Hierarchical Air Conditioning System

<u>Area</u>: Group of buildings
<u>Building</u>: Set of floors
<u>Floor</u>: Set of rooms

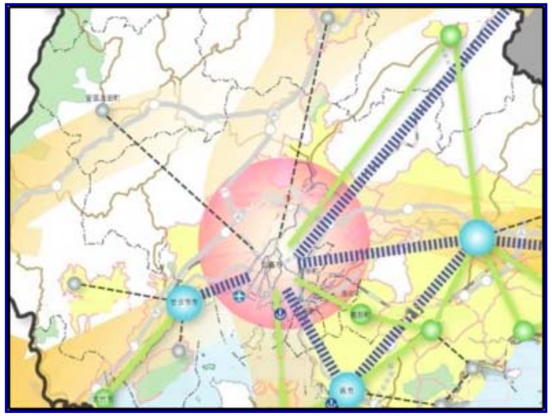
Urban Heat Island Problem

Local Actions of Measurement & Control

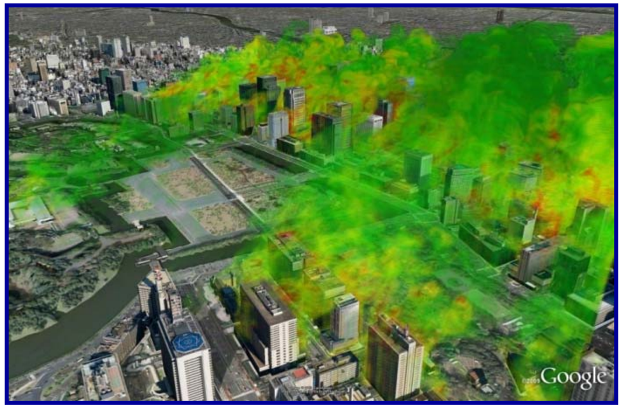
Realization of Global Desired Environment of a Whole City



Scale of buildings and roads



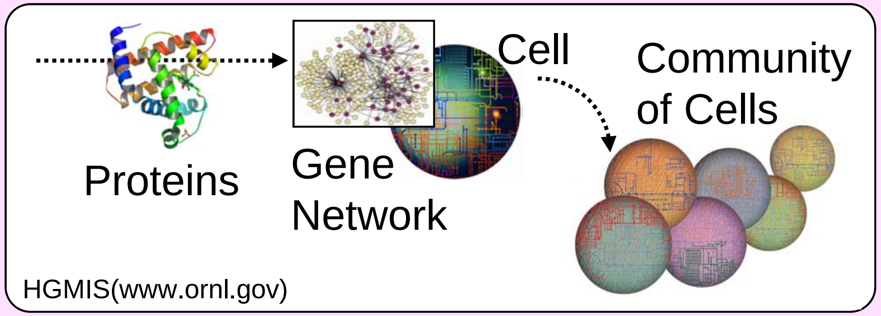
Glocal Control



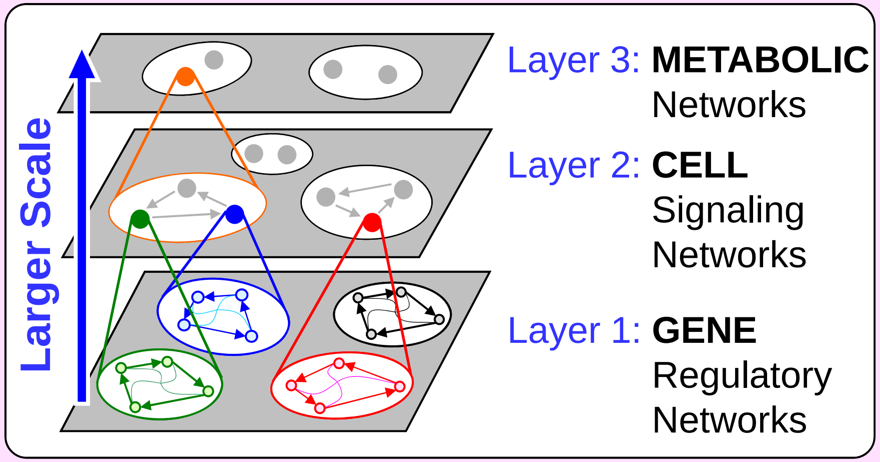
Scale of residential and business areas

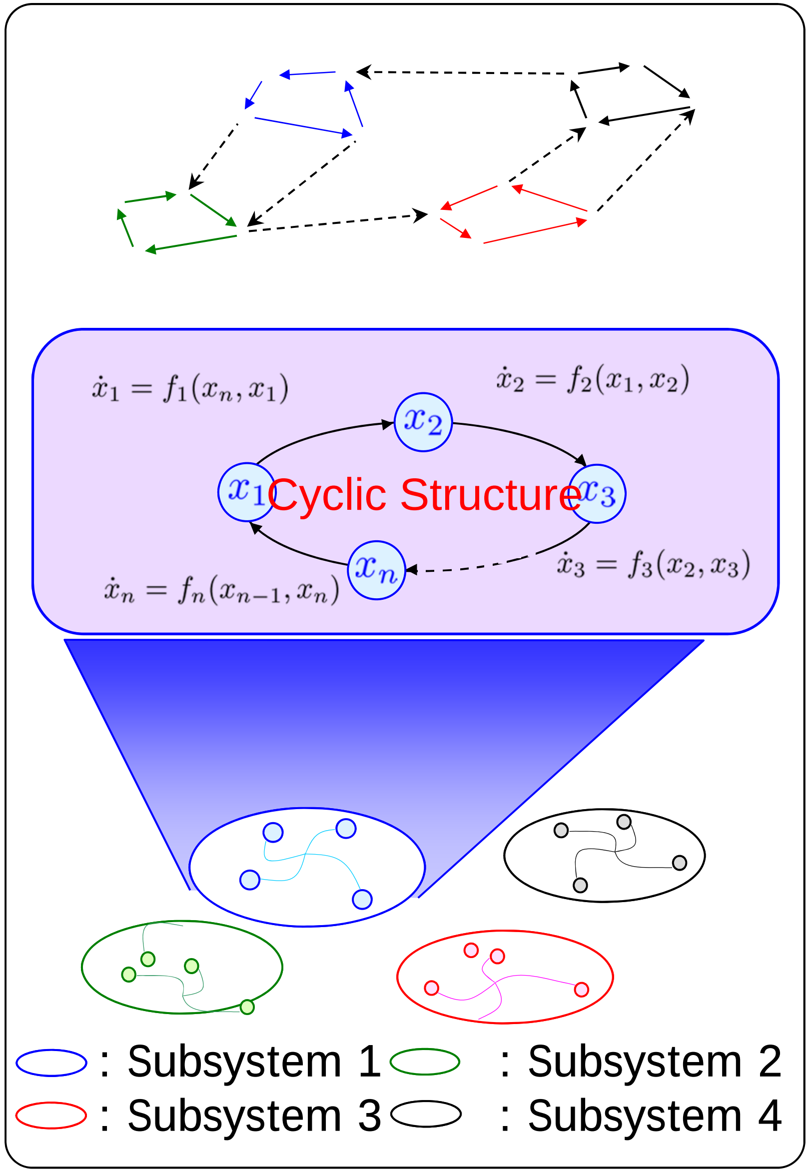
Scale of districts/towns

Hierarchical Bio-Network Systems



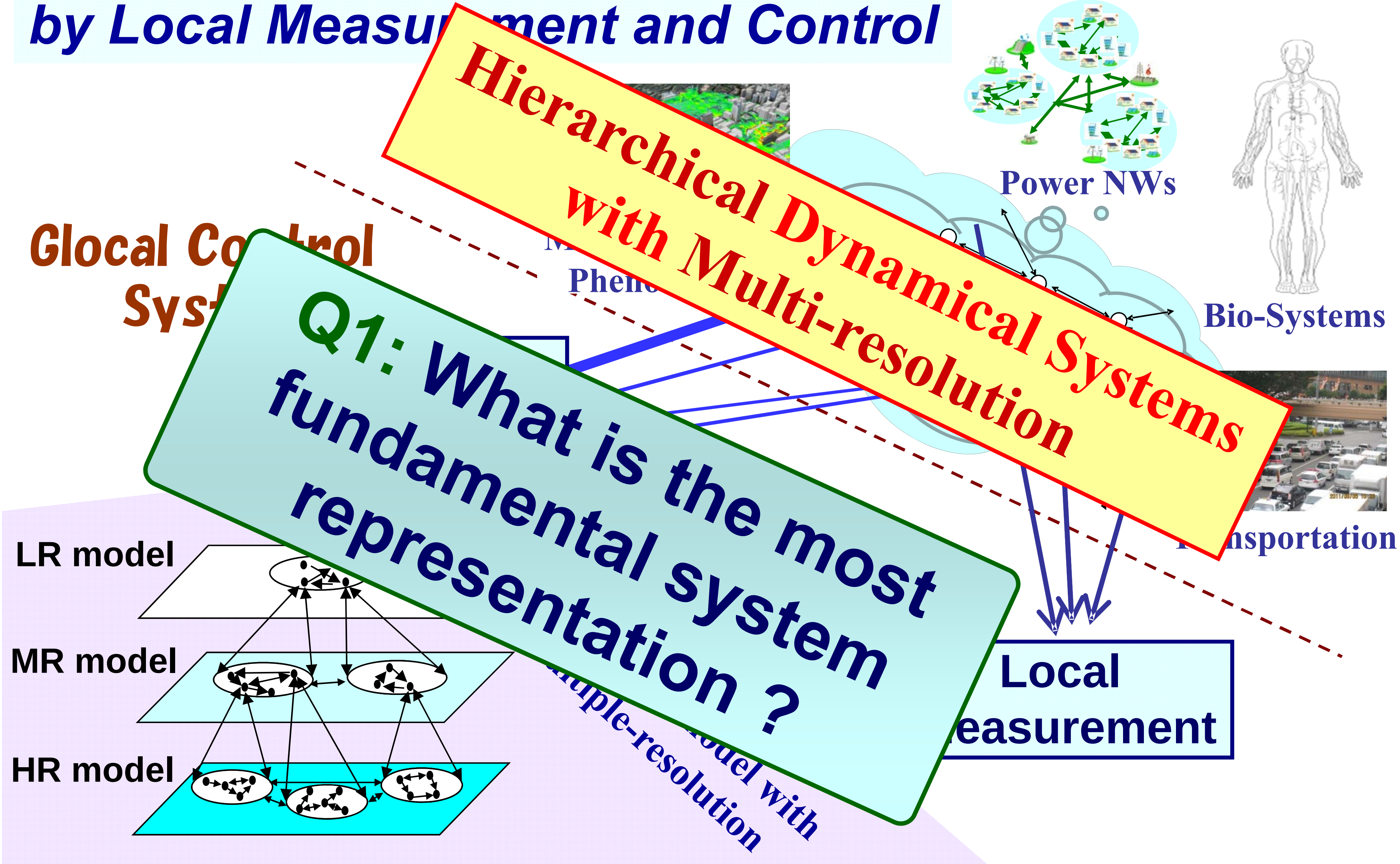
Hierarchical
Bio-Network Systems





Framework for Glocal Control

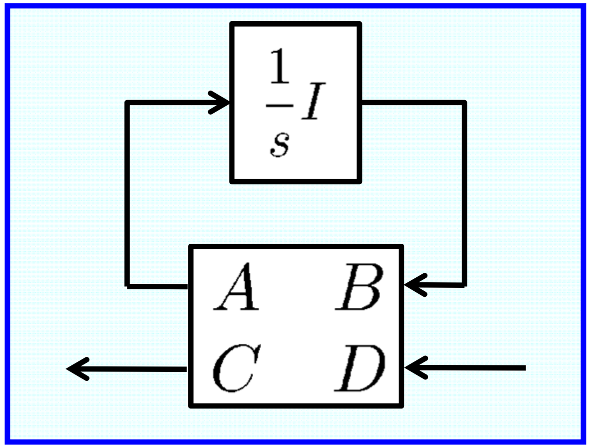
Realization of Global Functions by Local Measurement and Control



LTI System with Generalized Frequency Variable

A unified representation for homogeneous multi-agent dynamical systems

$C\,(sI - A)^{-1}\,B + D$

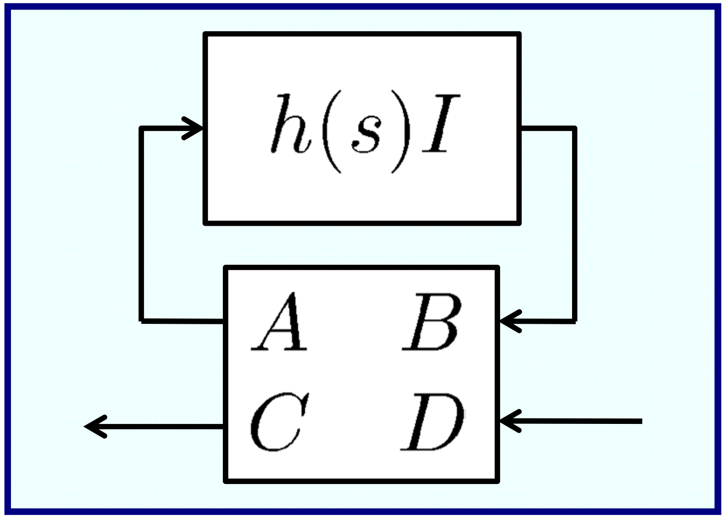


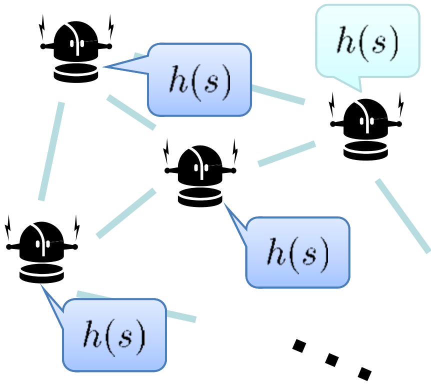
1/s → h(s)

Φ(s) = 1/h(s)

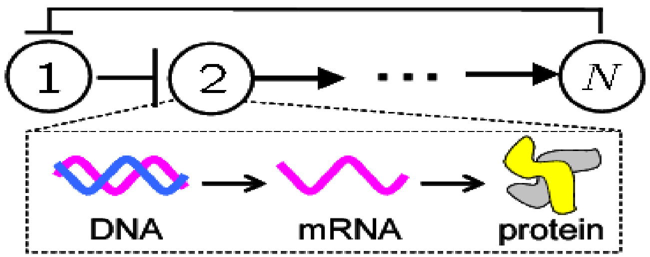
Generalized Freq. Variable

$C\,(\phi(s)I - A)^{-1}\,B + D$





Group Robot



Gene Reg. Networks

Dynamics + Information Structure

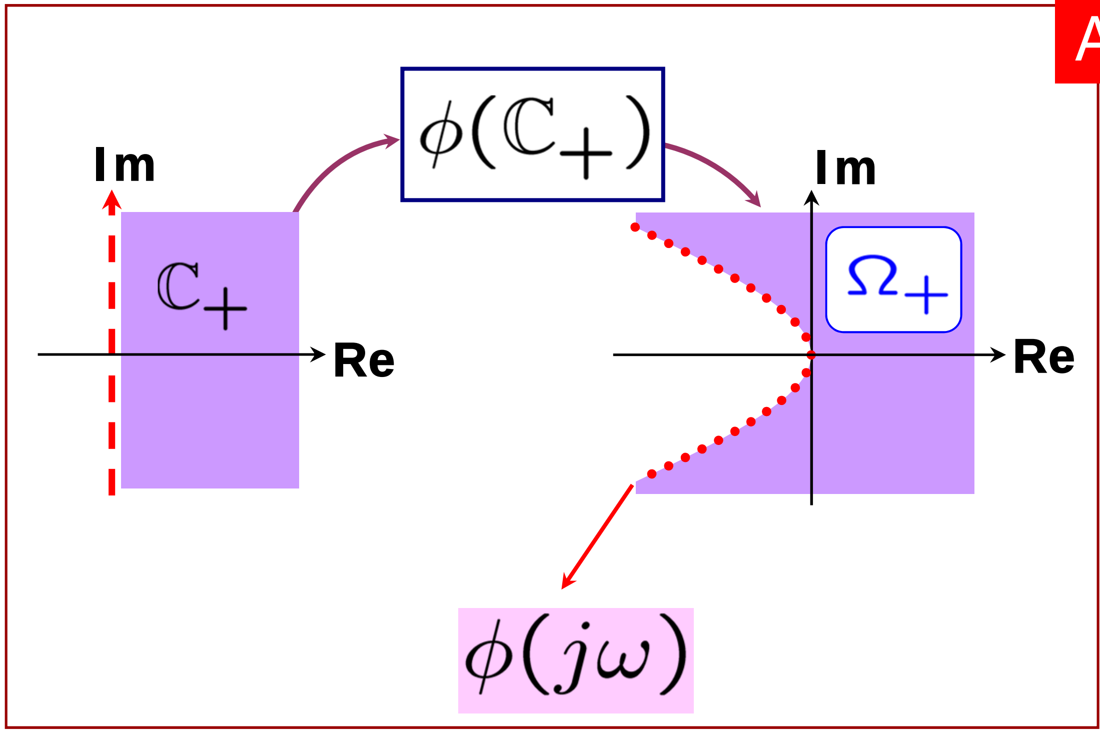
OUTLINE

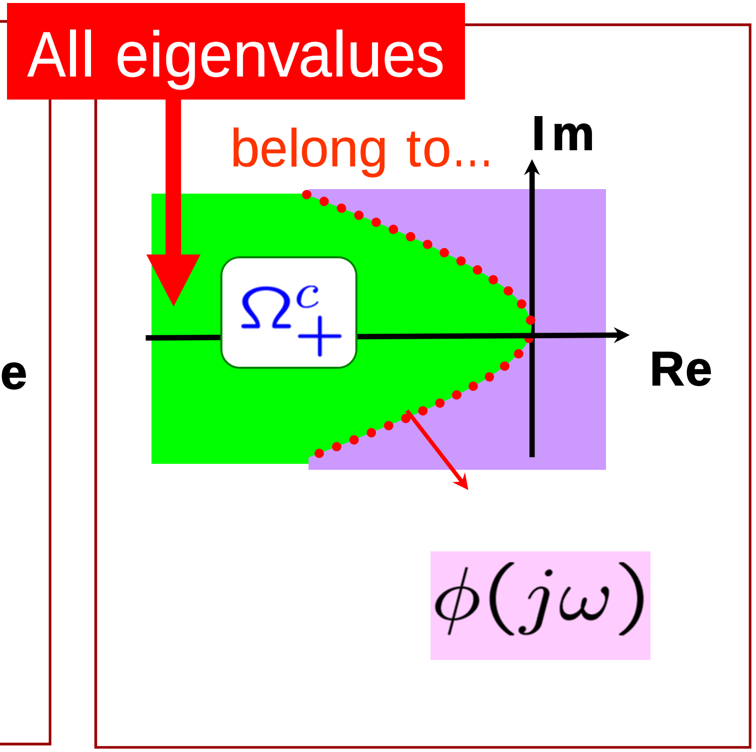
1. Stability Analysis: Review
2. D-Stability Analysis
3. Robust Stability Analysis
4. Application to Gene Regulatory Networks
5. Nonlinear Stability Analysis
6. Concluding Remarks

Stability Region for LTISwGFV

(Hara et al. IEEE CDC, 2007)

❖ <u>Define</u>: Domains $\Omega_+ := \phi(\mathbb{C}_+)$, $\Omega_+^c := \mathbb{C}\backslash\Omega_+$





Q2A: *How to characterize the region ?*
Q2B: *How to check the condition ?*

Stability Tests for LTISwGFV

Graphical	Algebraic	Numeric (LMI)
Nyquist – type	Hurwitz – type	Lyapunov – type
Fax & Murray (2004) Hara et al. (2007)	**Tanaka, Hara, Iwasaki (2009)**	**Tanaka, Hara, Iwasaki (2009)**
$h(s)$ and $\sigma(A)$	$h(s)$ and $\sigma(A)$	$h(s)$ and A
Characteristic Polynomial	Hurwitz test for complex coefficients	Generalized Lyapunov Ineq.

$p(\lambda, s) := d(s) - \lambda\, n(s)$ $\quad \lambda \in \sigma(A)$ **(complex)**

Stability Conditions

(Tanaka et al., ASCC, 2009)

Given $h(s) = n(s)/d(s),\ A$ $\boxed{\mathcal{H}_A(s) \text{ is stable}}$

Key lemma

$$\boxed{\sigma(A) \subset \Lambda := \{\ \lambda \in \mathbb{C} \mid d(s) - \lambda n(s) \text{ is Hurwitz stable}\ \}}$$

Extended Routh-Hurwitz Criterion [Frank,1946]

Algebraic condition

$$\sigma(A) \subset \bigcap_{k=1}^{\nu} \Sigma_k$$

$$\Sigma_k := \{\ \lambda \in \mathbb{C} \mid l_k(\lambda)^* \Phi_k l_k(\lambda) > 0\ \}$$

$$(k = 1, 2, \ldots, \nu)$$

Generalized Lyapunov inequality

$$l_\ell(\lambda) := \begin{bmatrix} 1 \\ \lambda \\ \vdots \\ \lambda^\ell \end{bmatrix},\quad L_\ell(A) := \begin{bmatrix} I \\ A \\ \vdots \\ A^\ell \end{bmatrix}$$

LMI feasibility problem

$X_k = X_k^T > 0$ s.t. $L_k(A)^T(\Phi_k \otimes X_k)L_k(A) > 0$

for each $k = 1, 2, \ldots, \nu$

Numerical Example : 2nd order (1/2)

Given $h(s) = \dfrac{2s+1}{s^2+s+1}$, $A \in \mathbb{R}^{n\times n}$

$$\sigma(A) \subset \Lambda := \{\ \lambda \in \mathbb{C} \mid (s^2+s+1) - \lambda(2s+1) \text{ is Hurwitz stable }\}$$

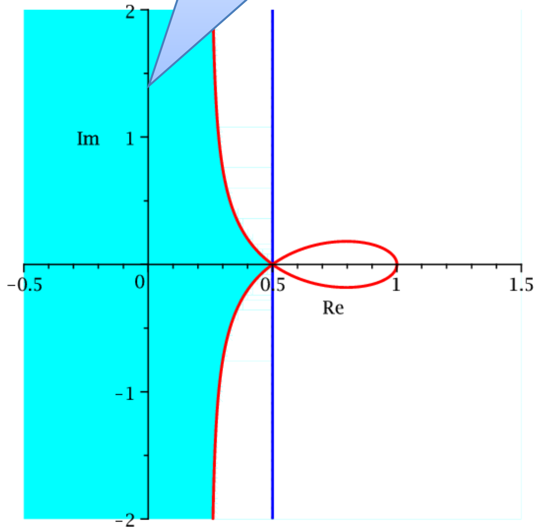
Extended Routh-Hurwitz Criterion (Frank, 1948)

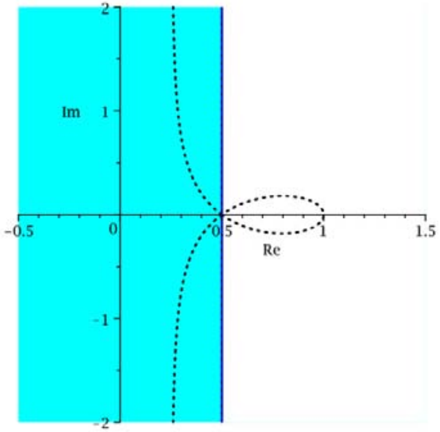
$$\sigma(A) \subset \Sigma := \{\ \lambda \in \mathbb{C} \mid \Delta_1 > 0 \text{ and } \Delta_2 > 0\ \}$$

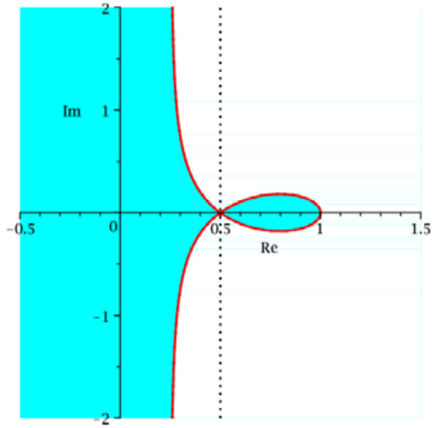
$$\Delta_1 = \begin{bmatrix} 1 \\ \lambda \end{bmatrix}^* \begin{bmatrix} 1 & -1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ \lambda \end{bmatrix} > 0$$

$$\Delta_2 = \frac{1}{4} \begin{bmatrix} 1 \\ \lambda \\ \lambda^2 \end{bmatrix}^* \begin{bmatrix} 4 & -10 & 7 \\ -10 & 18 & -8 \\ 7 & -8 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ \lambda \\ \lambda^2 \end{bmatrix} > 0$$

Stability region







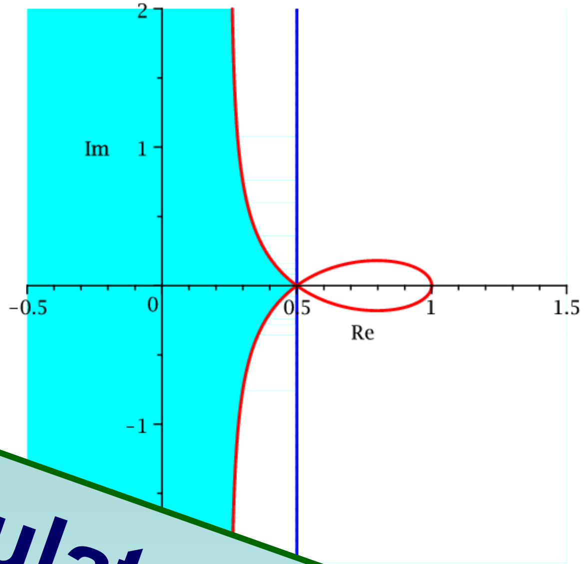
Numerical Example : 2^{nd} order (2/2)

Given $h(s) = \dfrac{2s+1}{s^2+s+1}$, $A \in \mathbb{R}^{n\times n}$

$\sigma(A) \subset$ … $\mid$ $\Delta_1 > 0$ and $\Delta_2 > 0$ $\}$

$\Delta_1 =$ …

$\Delta_2 = \frac{1}{4}\begin{bmatrix} \\ \lambda \\ \lambda^2 \end{bmatrix}$ …



Q3: How can we calculate Φ_k ?
Is there any systematic way for doing it ?

Generalized Ly…

$$X_1 = X_1^T > 0 \quad \text{s.t.} \quad \begin{bmatrix} I \\ A \end{bmatrix}^T \left(\begin{bmatrix} 1 & -1 \\ -1 & 0 \end{bmatrix} \otimes X_1 \right) \begin{bmatrix} I \\ A \end{bmatrix} > 0$$

$$X_2 = X_2^T > 0 \quad \text{s.t.} \quad \begin{bmatrix} I \\ A \\ A^2 \end{bmatrix}^T \left(\begin{bmatrix} 4 & -10 & 7 \\ -10 & 18 & -8 \\ 7 & -8 & 0 \end{bmatrix} \otimes X_2 \right) \begin{bmatrix} I \\ A \\ A^2 \end{bmatrix} > 0$$

Algorithm

Given Data: all coefficients of numerator and denominator of *h(s)*

Algorithm h2Phi($h(s)$) :

Input : $h(s) = \dfrac{b_1 s^{\nu-1} + \cdots + b_\nu}{s^\nu + a_1 s^{\nu-1} + \cdots + a_\nu}$

Output : ℓ_k and Φ_k

1. $p_0 \leftarrow 1,\ q_0 \leftarrow 0$
   for $i \leftarrow 1$ **until** $2\nu - 1$ **do**
   if $i \le \nu$ **then**
   $p_i \leftarrow a_i - b_i x,\ q_i \leftarrow -b_i y$
   else
   $p_i \leftarrow 0,\ q_i \leftarrow 0$

2 $\Delta_1 \leftarrow p_1$
for $k \leftarrow 2$ **until** 2ν **do**
$M \leftarrow O^{(2k-1)\times(2k-1)}$
for $i \leftarrow 1$ **unt**
if i **is**

$-2)/2$ **do**
$\leftarrow p_{2k-i-2(m-1)}$
until $k-(i-2)/2-1$ **do**
$(i, 2k-m) \leftarrow p_{2k-i-1-2(m-1)}$
$M^{k+1,k+2,\ldots,2k-1}_{2,3,\ldots,k} \leftarrow -M^{1,2,\ldots,k-1}_{k+1,k+2,\ldots,2k-1}$
$M^{k+1,k+2,\ldots,2k-1}_{k+1,k+2,\ldots,2k-1} \leftarrow M^{1,2,\ldots,k-1}_{1,2,\ldots,k-1}$
$\Delta_k(x,y) \leftarrow |M|$

1 **until** ν **do**
$\leftarrow \Delta_k((\lambda+\bar\lambda)/2, (\lambda-\bar\lambda)/2j)$
ntil ν **do**
imum of the degree of λ in $\Delta'_k(\lambda,\bar\lambda)$ -1

ntil $\ell_k - 1$ **do**
$\leftarrow 0$ **until** $\ell_k - 1$ **do**
$\Phi_k(m+1, l+1)$
$\leftarrow$ the coefficient of $\lambda^m \bar\lambda^l$ in $\Delta'_k(\lambda,\bar\lambda)$

5 **rerurn** ℓ_k and Φ_k.

Systematic methods for stability analysis

Hurwitz-type & LMI

Result:

$$\Phi_k \quad (k = 1, 2, \cdots, \nu)$$

Numerical Example : 4th order

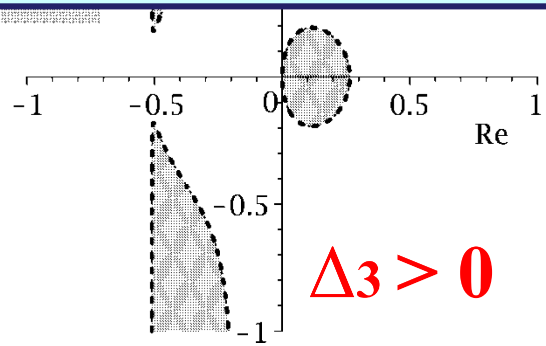
$$h(s) = \frac{100(s+2)(\frac{19}{10}s^2 - \frac{1}{500000}s + \frac{21}{10})}{(s-1)^2(s+1)(s+100)}$$

Unstable
Non-minimum phase

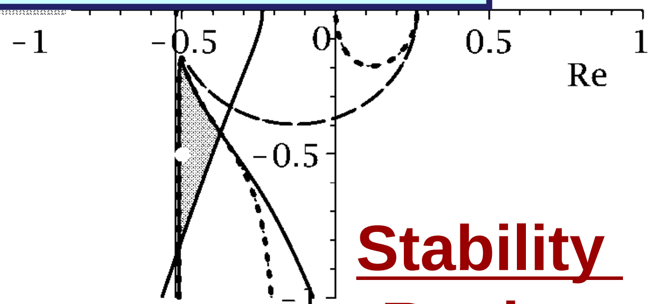
Δ1

Summary 1: LTI System with GFV

- **One of nice classes for NDSs**
- **Stability and robust stability conditions**
- **Many application fields**
  - **synchronization of coupled oscillators**
  - **bio-systems**



$\Delta_3 > 0$



Stability Region

Further Fundamental Questions

Robustness Issues in
Networked Dynamical Systems
(LTI systems with GFV) ***?***

Q4 : *D-Stability Analysis ?*

Stability Margins

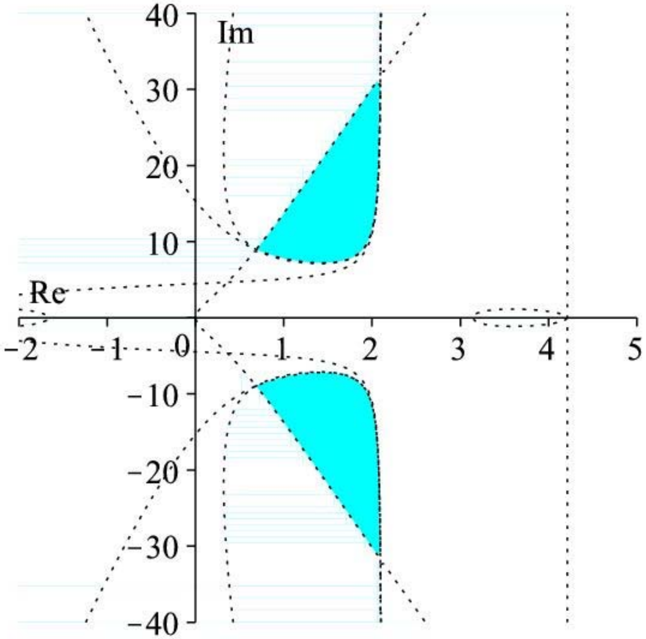
Q5 : *Robust Stability Analysis ?*

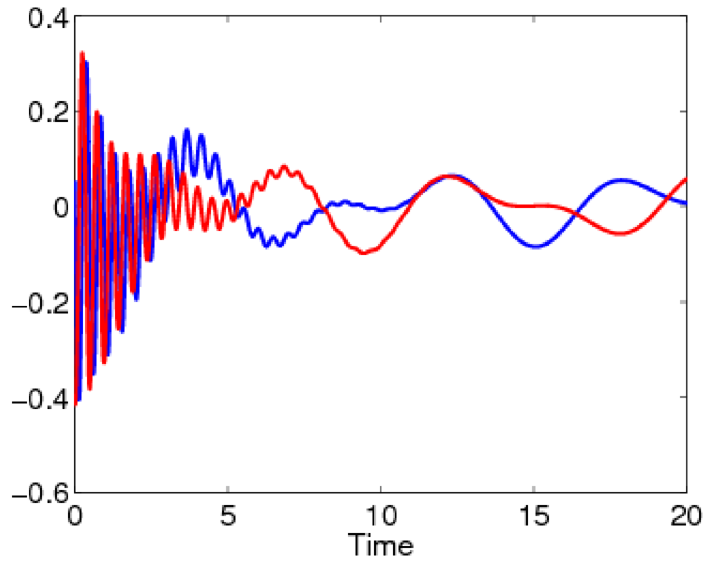
Homogeneous → Heterogeneous

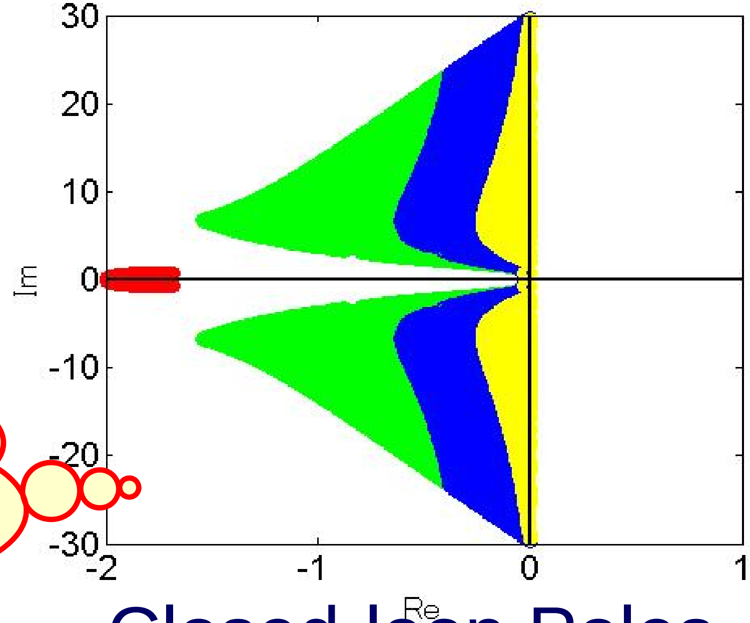
OUTLINE

Why D-Stability Analysis ?

$$h(s) = \frac{(\frac{1}{2}s+1)(\frac{19}{10}s^2 - \frac{1}{500}s + \frac{21}{10})}{s(s-2)(s+1)(s+5)}$$



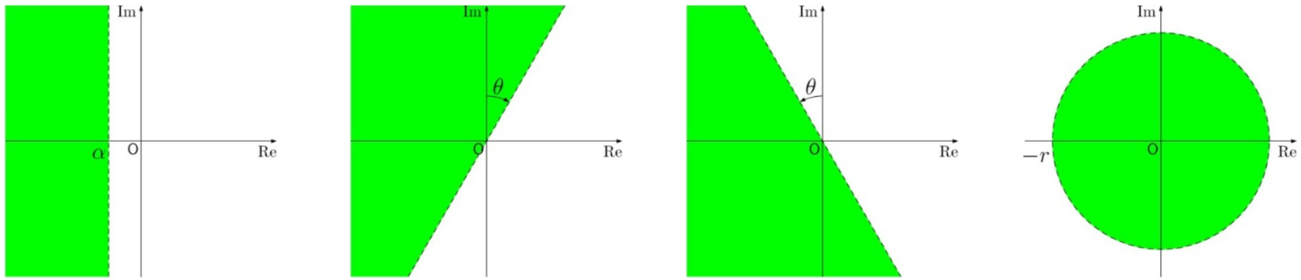


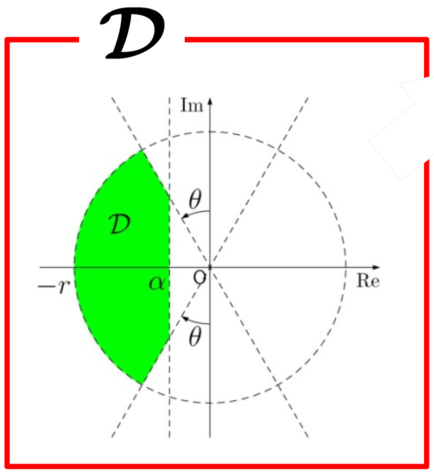


Closed-loop Poles

We can not assign all the closed-loop poles as you want.

Unified Approach to D-Stability Analysis





Unified Stability Analysis for Disks and Half Planes

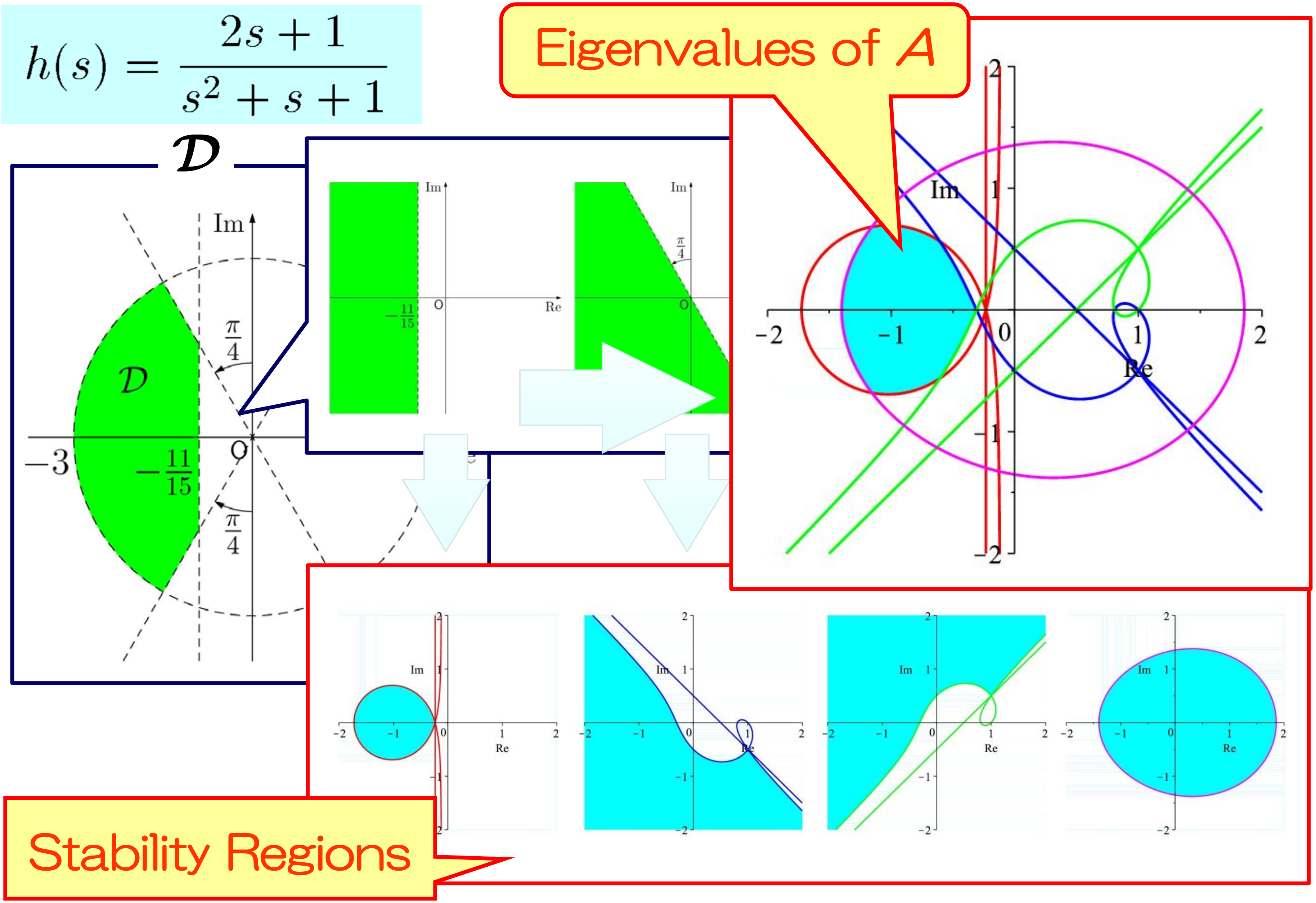
Derive the stability conditions

Intersection

D-stability Condition

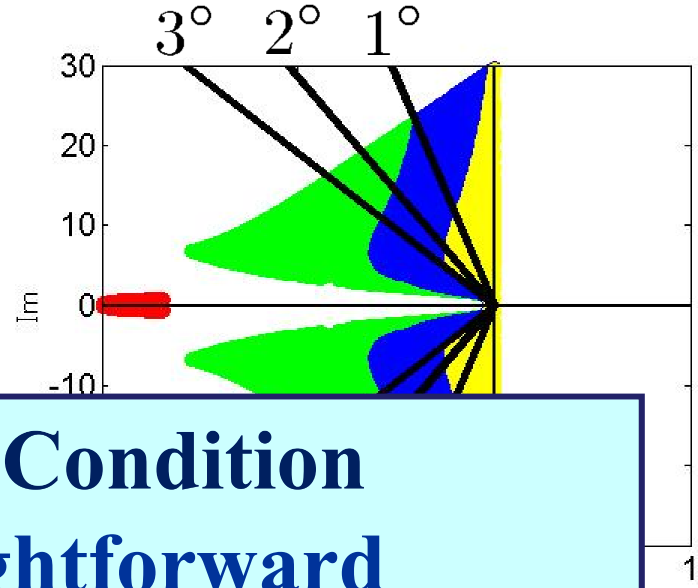
A Numerical Example

$$h(s) = \frac{2s+1}{s^2+s+1}$$



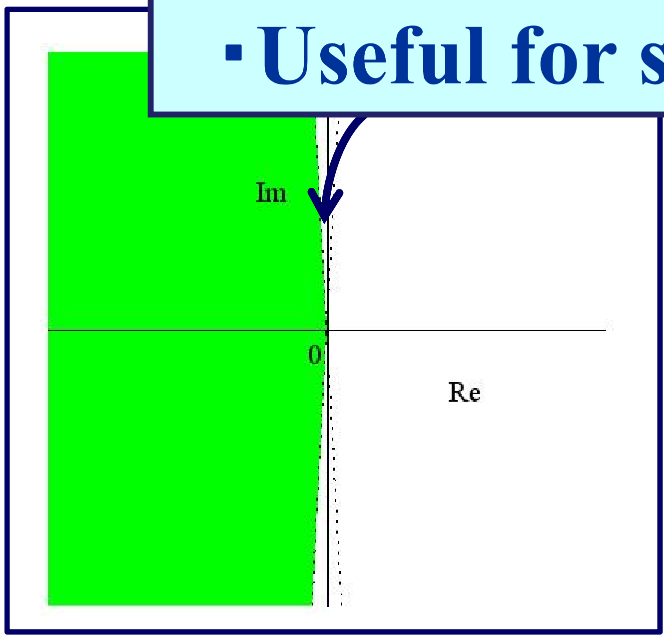
Motivating Example

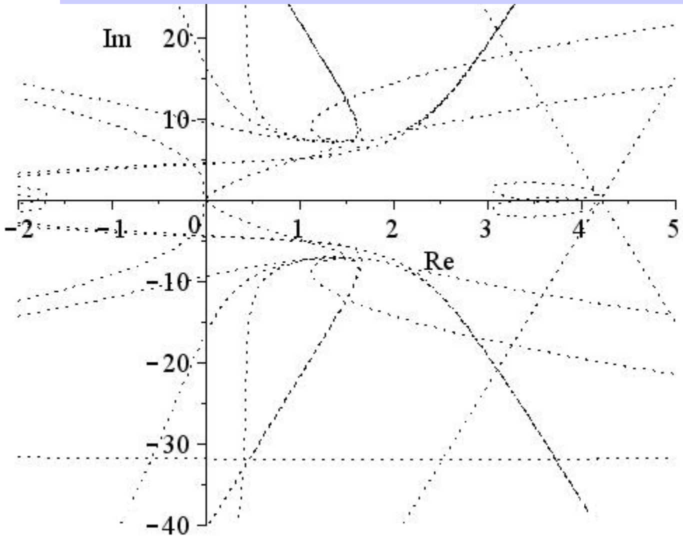
$$h(s) = \frac{(\frac{1}{2}s+1)(\frac{19}{10}s^2 - \frac{1}{500}s + \frac{21}{10})}{s(s-2)(s+1)(s+5)}$$



Summary 2 : D-Stability Condition

- **Complicated but straightforward**
- **Useful for some control performances**



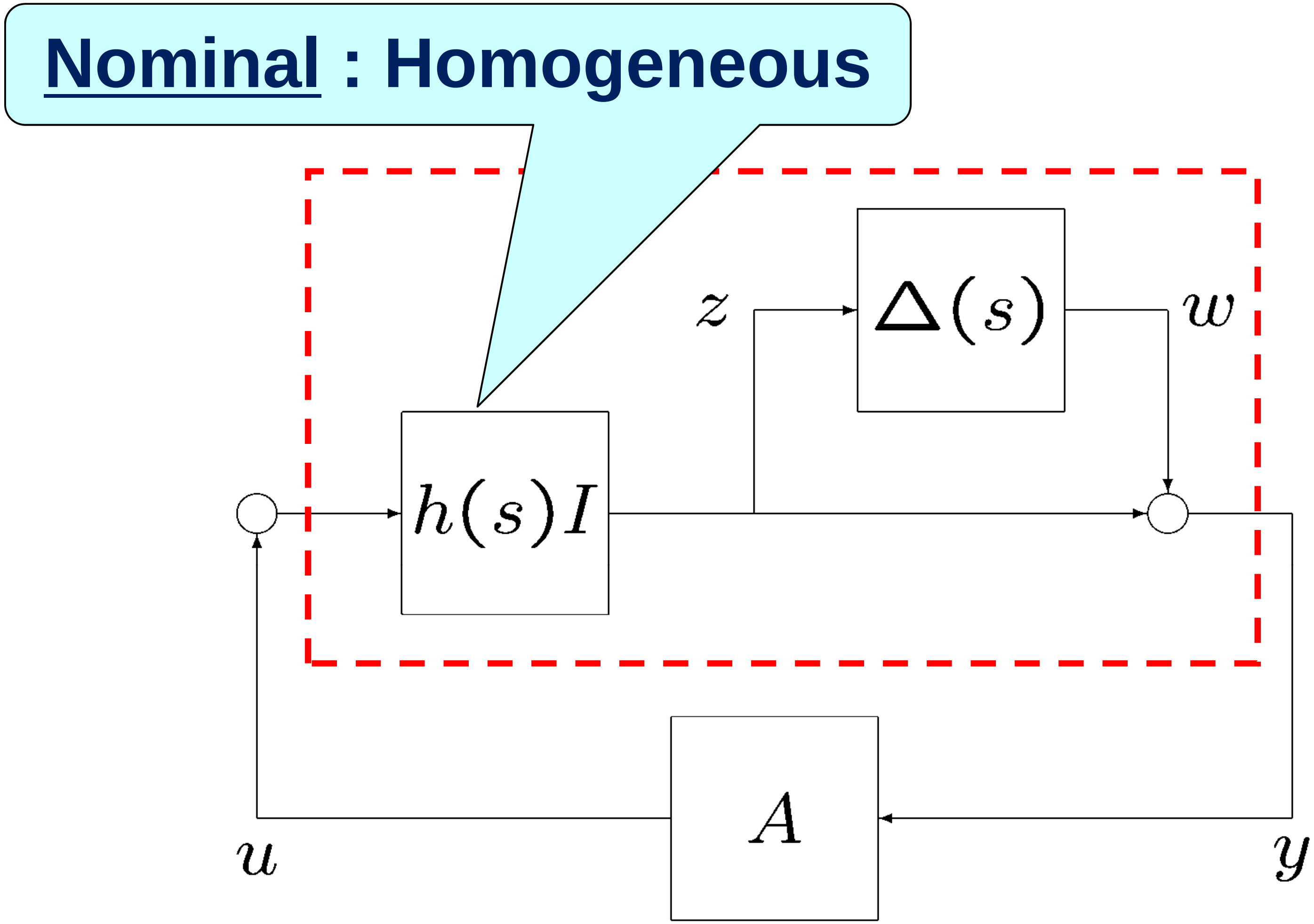


OUTLINE

1. Stability Analysis: Review
2. D-Stability Analysis
3. **Robust Stability Analysis**
4. Application to Gene Regulatory Networks
5. Nonlinear Stability Analysis
6. Conclusion

Multiplicative Perturbations

$$\tilde{H}(s) = (I + \Delta(s)) \cdot h(s)$$



Three Classes of Perturbations

Multiplicative Perturbation:

$$\tilde{H}(s) = (I + \Delta(s)) \cdot h(s)$$

Three Classes:

Full perturbation:

$$\mathbf{\Delta}_\gamma \ := \ \{\, \Delta(s) \in \mathbf{\Delta}_p \mid \ \|\Delta\|_\infty \le 1/\gamma \,\}$$

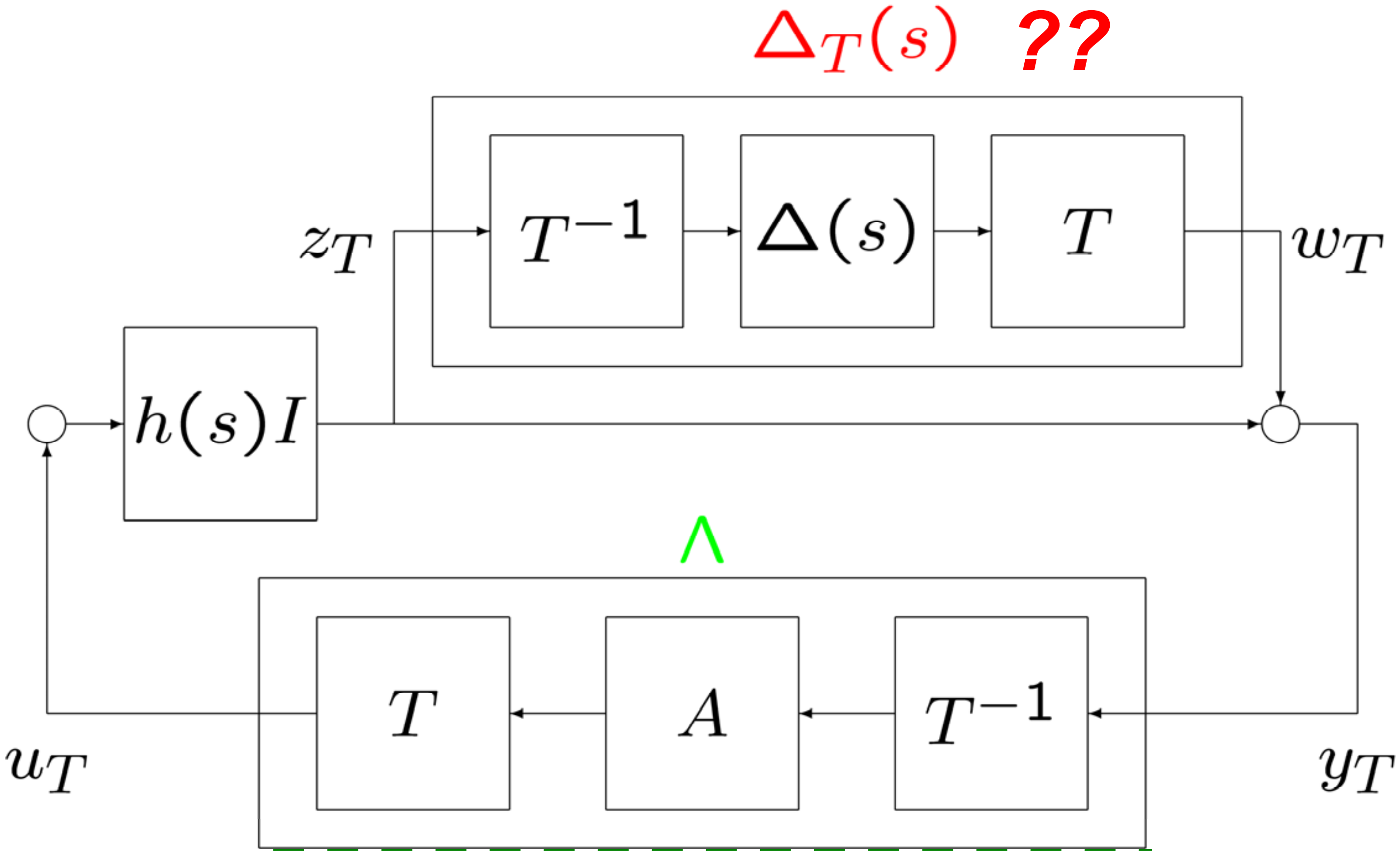
Heterogeneous:

$$\mathbf{\Delta}_{d\gamma} \ := \ \{\, \Delta(s) \in \mathbf{\Delta}_\gamma \mid \Delta(s) \ : \ \text{diagonal} \,\}$$

Homogeneous:

$$\mathbf{\Delta}_{I\gamma} \ := \ \{\, \Delta(s) \in \mathbf{\Delta}_\gamma \mid \Delta(s) = \delta(s) I \,\}$$

Basic Idea

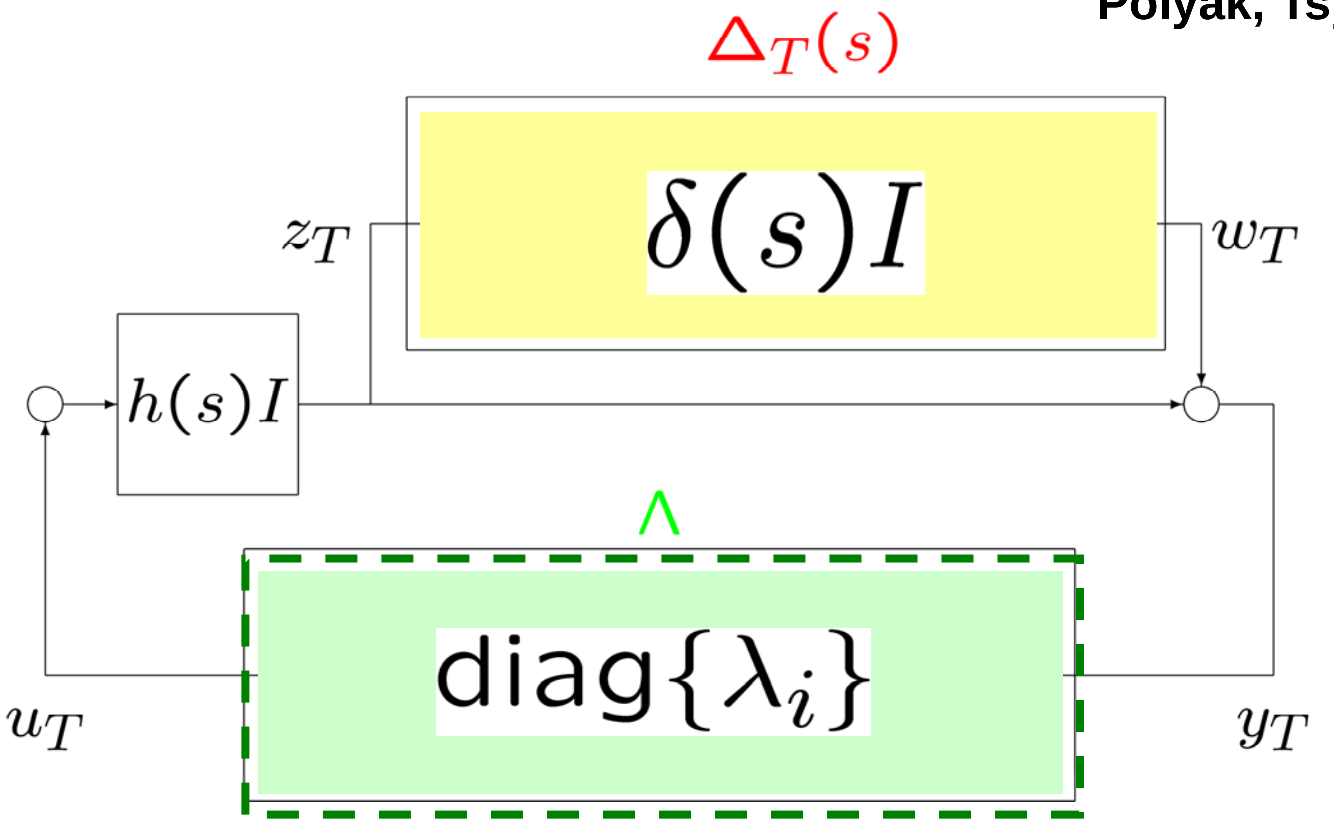


$$\Lambda := TAT^{-1} = \mathrm{diag}\{\lambda_i\}$$

***A*: diagonalizable**

Homogeneous Perturbations

Polyak, Tsypkin (1996)



$$w_T \rightarrow z_T \ : \ \mathrm{diag}\{\frac{\lambda_i h(s)}{1-\lambda_i h(s)}\}$$

Complementary Sensitivity function (*h(s), λi*)

Robust Stability Condition for Homogeneous Perturbations

$$\tilde{H}(s) = (1 + \delta(s)) \cdot h(s) I$$

Small Gain Criterion

Theorem: The following three conditions are equivalent.

(i) The system is robustly stable for $\mathbf{\Delta}_{I\gamma}$.

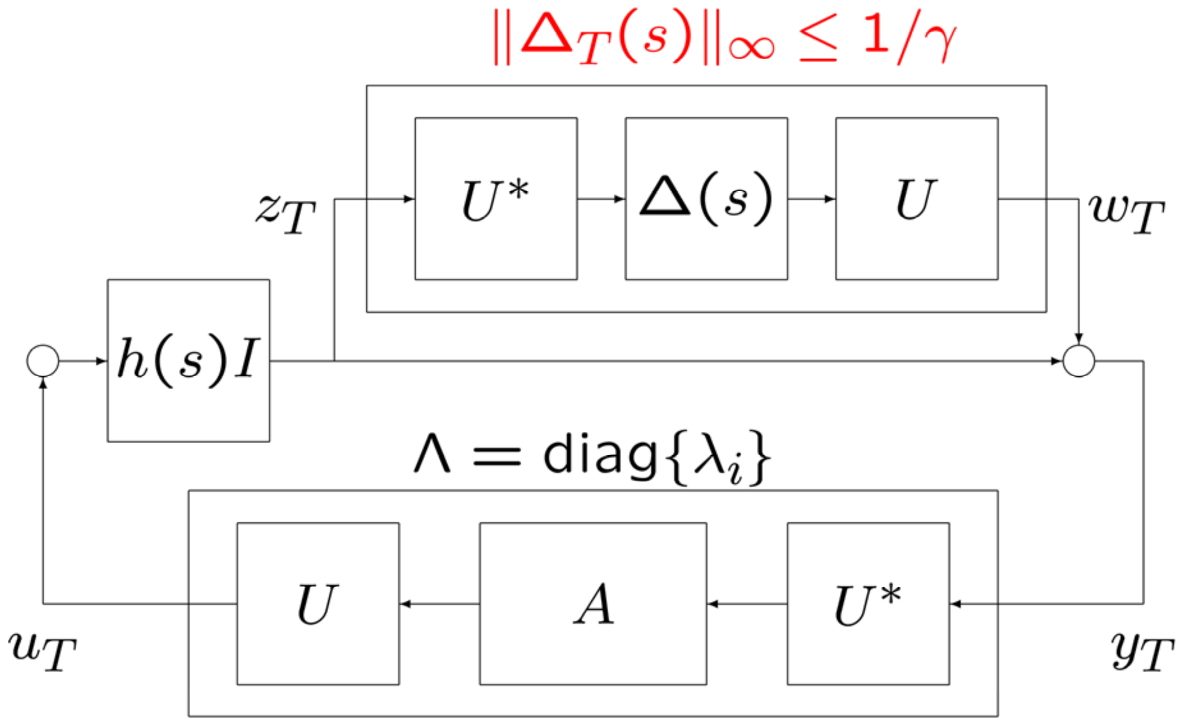
(ii) $\left\| \dfrac{\lambda h}{1 - \lambda h} \right\|_\infty < \gamma, \quad \forall\ \lambda \in \sigma(A)$

(iii) $\left| \dfrac{\lambda}{\phi - \lambda} \right| < \gamma, \quad \forall\ \lambda \in \sigma(A),$

$$\forall\ \phi \in \mathbf{\Phi} := \{1/h(j\omega) |\ \omega \in \mathbb{R}\ \}.$$

A : Normal ($T = U$: Unitary Matrix)

$A \in \mathbb{R}^{n\times n}$ is normal, i.e., $A^TA = AA^T$.



* Symmetric
* Skew - Symmetric
* Circulant

Sufficiency: small gain condition

Necessity: worst case $\Delta(s) = \delta(s)I$

Robust Stability Condition for Full Perturbations

Hara, Tanaka, Iwasaki (ACC2010)

Assumption

$A \in \mathbb{R}^{n\times n}$ is normal, i.e., $A^TA = AA^T$.

Theorem: The following three conditions are equivalent.

(i) The system is robustly stable for $\mathbf{\Delta}_\gamma$.

(ii) $\left\| \dfrac{\lambda h}{1-\lambda h} \right\|_\infty < \gamma, \quad \forall\ \lambda \in \sigma(A)$

(iii) $\left| \dfrac{\lambda}{\phi - \lambda} \right| < \gamma, \quad \forall\ \lambda \in \sigma(A),$

$$\forall\ \phi \in \mathbf{\Phi} := \{1/h(j\omega) |\ \omega \in \mathbb{R}\ \}.$$

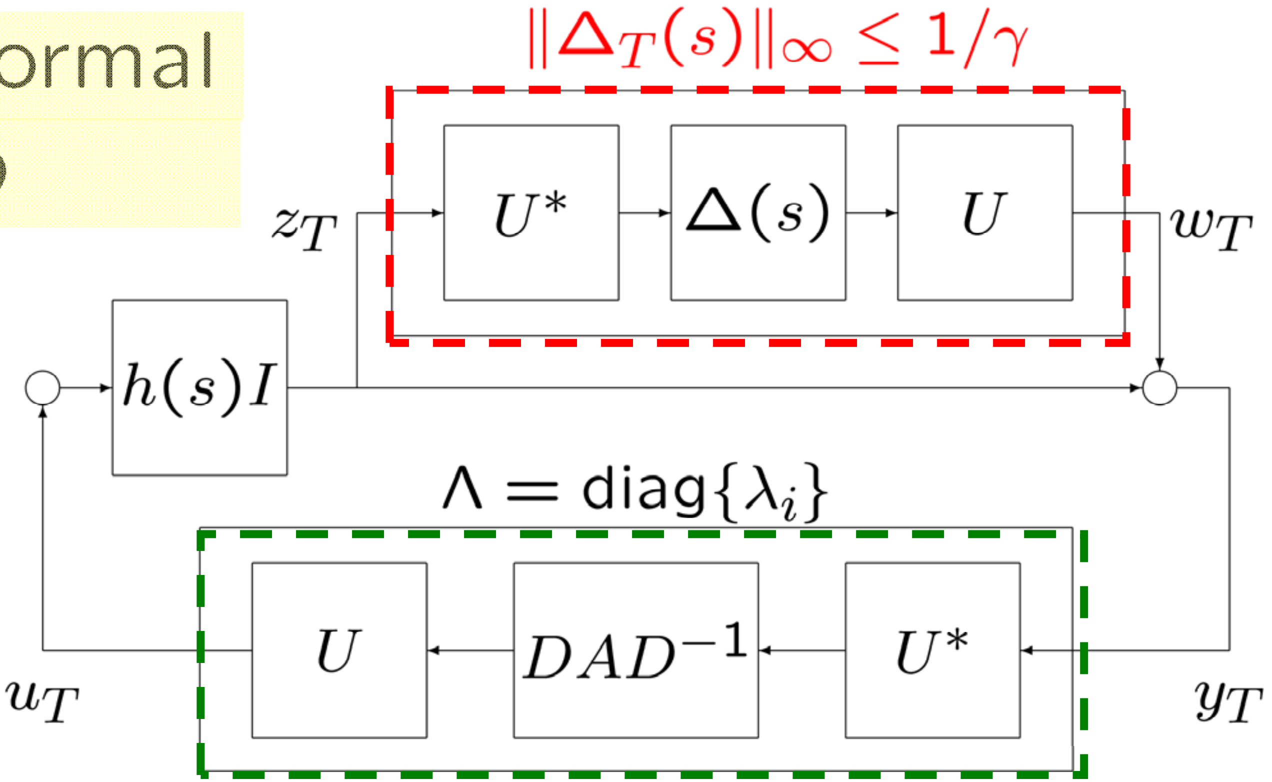
Heterogeneous Perturbations

$$\Delta(s) = \mathrm{diag}\{\delta_i(s)\}$$

$$^{\forall} D = \mathrm{diag}\{d_i\} > 0 \text{ s.t. } D\Delta(s)D^{-1} = \Delta(s)$$

DAD^{-1} is normal

$$T = UD$$



Robust Stability Condition for Heterogeneous Perturbations

Assumption

$^{\exists}D$: diagonal s.t. DAD^{-1} is normal

Theorem: The following three conditions are equivalent.

(i) The system is robustly stable for $\mathbf{\Delta}_{d\gamma}$.

(ii) $$\left\|\frac{\lambda h}{1-\lambda h}\right\|_\infty < \gamma, \quad \forall\ \lambda \in \sigma(A)$$

(iii) $$\left|\frac{\lambda}{\phi-\lambda}\right| < \gamma, \quad \forall\ \lambda \in \sigma(A),$$

$$\forall\ \phi \in \mathbf{\Phi} := \{1/h(j\omega) |\ \omega \in \mathbb{R}\ \}.$$

Coprime Factor Perturbations (1/2)

$$G(s) := \begin{bmatrix} A \\ I \end{bmatrix} (I - h(s)A)^{-1} \begin{bmatrix} h(s)I & I \end{bmatrix}$$

$$\Downarrow \quad A = U^* \Lambda U$$

$$G(s) = \begin{bmatrix} U^* & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} \Lambda \\ I \end{bmatrix} (I - h(s)\Lambda)^{-1} \begin{bmatrix} h(s)I & I \end{bmatrix} \begin{bmatrix} U & 0 \\ 0 & I \end{bmatrix}$$

$$\|G\|_\infty < \gamma \iff \left\| \begin{bmatrix} \lambda \\ 1 \end{bmatrix} (1 - h\lambda)^{-1} \begin{bmatrix} h & 1 \end{bmatrix} \right\|_\infty < \gamma, \quad \forall\ \lambda \in \sigma(A)$$

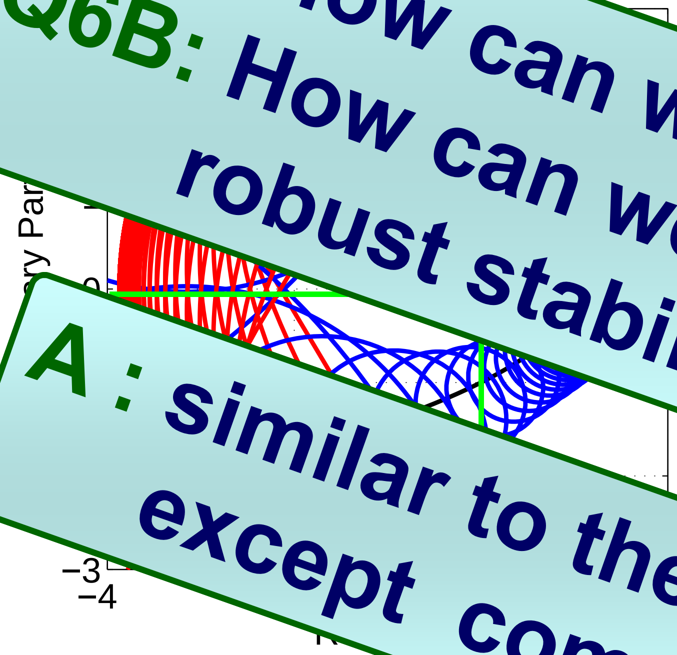
Coprime Factor Perturbations (2/2)

$$h(s) = \frac{1}{s^2 + s + 1}$$

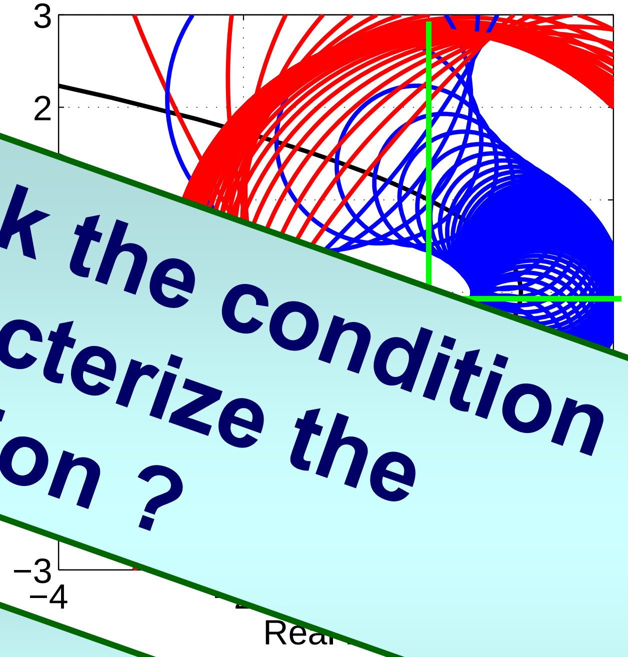
Q6A: How can we check the condition ?

Q6B: How can we characterize the robust stability region ?

A : similar to the standard cases except complex number λ



$\gamma = 4.0$



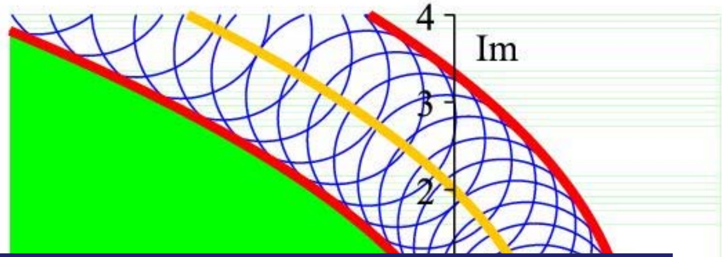
3.0

Outside of Blue Circle & Inside of … Circle

Analytic Expressions by QE

$$h(s) = 1/(s^2+2s+1)$$

$$\lambda = x + jy$$



Summary 3 : Robust Stability Conditions

- Class of *A* ⇔ Class of *Δ*
- Small gain (H∞-norm) condition is necessary & sufficient
- *D*-scaling technique is quite useful
- Some applications

  bio-systems

$$+ \quad (8x^3+6x^2-30x-1)y^2$$
$$+ \quad (16x^3-24x^2-15x-2)x > 0$$

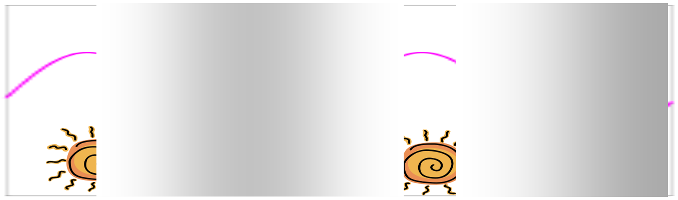
$$\begin{bmatrix} 0 & -\frac{1}{4} & 1 & \frac{21}{8} & -\frac{51}{4} & -\frac{7}{2} \\ \frac{1}{16} & -1 & \frac{27}{4} & -\frac{51}{4} & -8 & -1 \\ 0 & -\frac{1}{16} & \frac{3}{4} & -\frac{7}{2} & -1 & 0 \end{bmatrix}$$

OUTLINE

1. Stability Analysis: Review
2. D-Stability Analysis
3. Robust Stability Analysis
4. **Application to Gene Regulatory Networks**
5. Nonlinear Stability Analysis
6. Conclusion

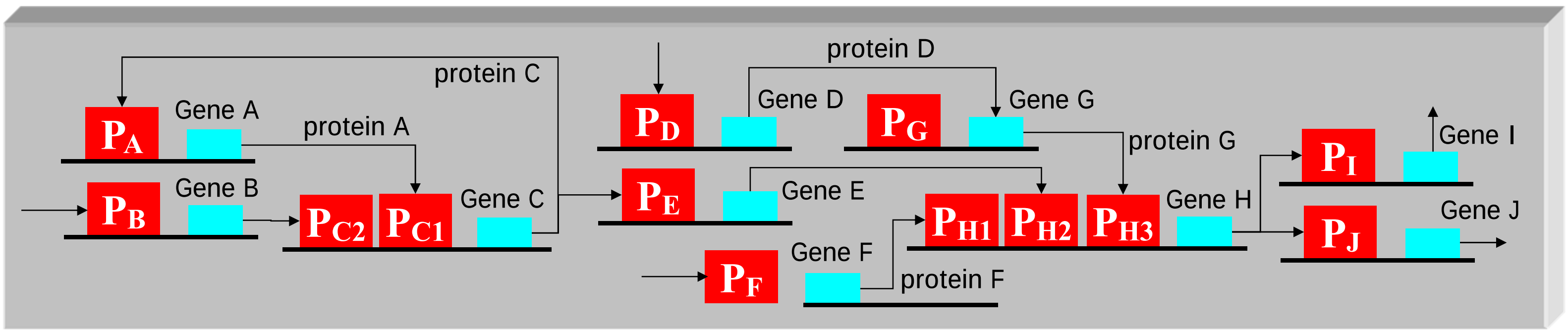
An Application：Biological rhythms

Motivation



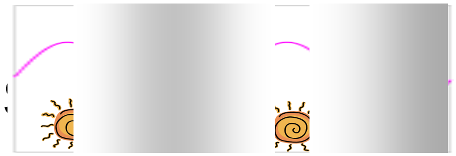
- **Biological rhythms**
  - 24h-cycle, heart beat, sleep cycle etc.
  - caused by periodic oscillations of protein concentrations in ***Gene Regulatory Networks***
- **Medical and engineering applications**
  - Artificially engineered biological oscillators

    (e.g.) Repressilator [Elowitz & Leibler, *Nature*, 2000]

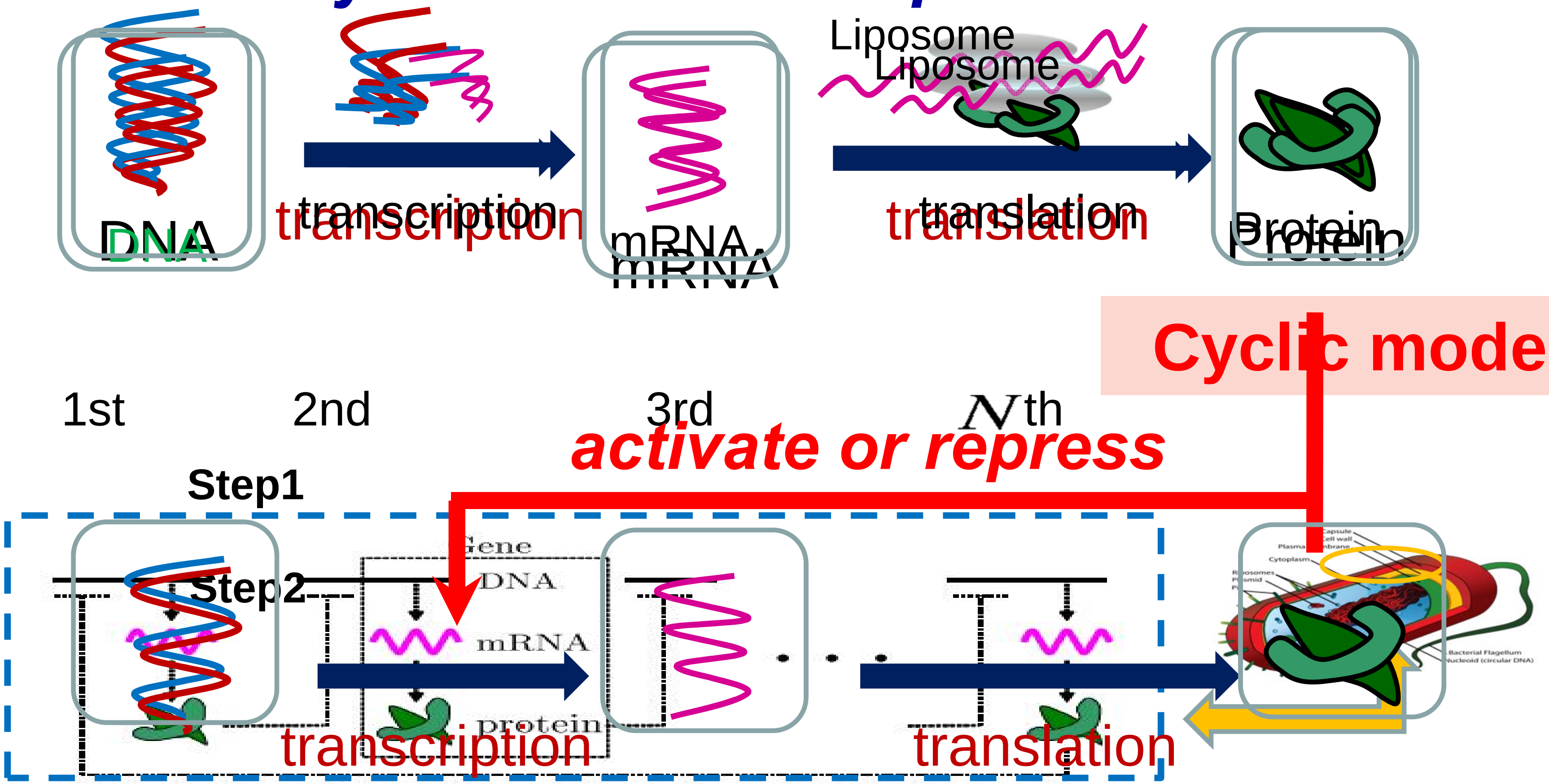


Gene Regulatory Network Systems

- **Biological rhythms:** 24h-cycle, heart beat
  periodic oscillations of protein concentrations
  in ***Gene Regulatory Networks***

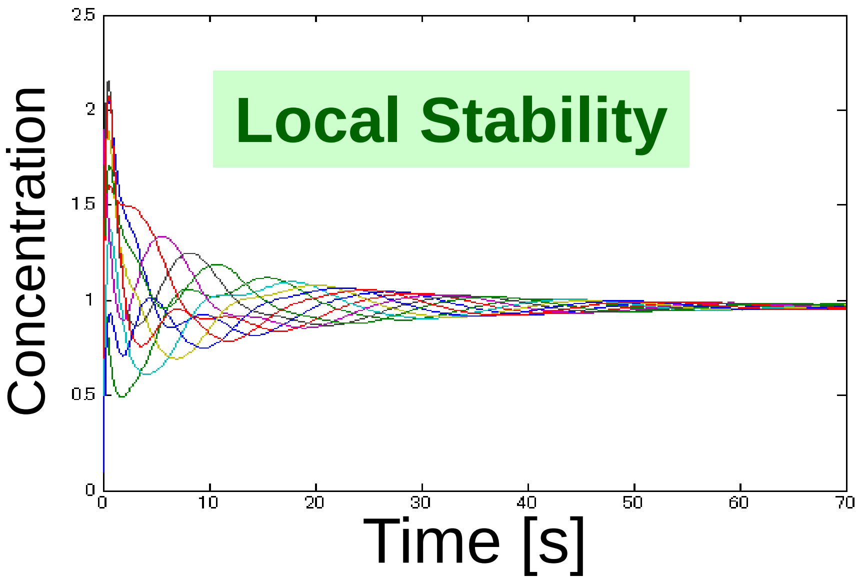


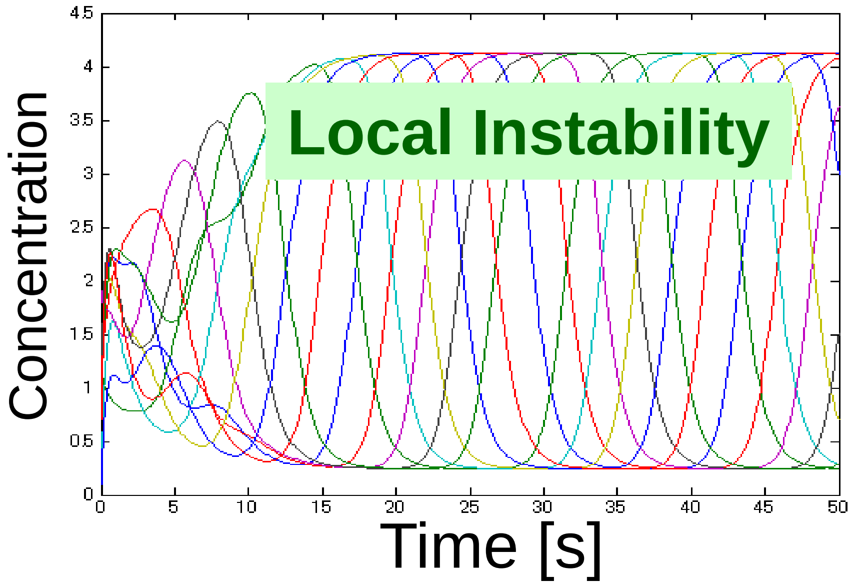
- ***Protein synthesis : transcription & translation***



Convergence or Oscillations ?

- **Numerical simulations**
  – Changing chemical parameters





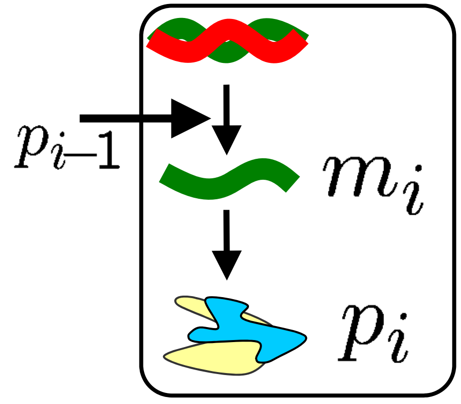
Q7: What are the conditions for convergence and the existence of oscillations ?

Nonlinear Analysis

Gene Regulatory Network Model

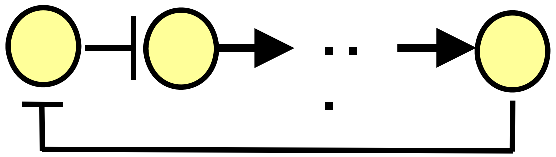
gene model $(i = 1, \cdots, N)$

$$\frac{d}{dt}\begin{bmatrix} r_i \\ p_i \end{bmatrix} = \begin{bmatrix} -a_i & 0 \\ c_i & -b_i \end{bmatrix}\begin{bmatrix} r_i \\ p_i \end{bmatrix} + \begin{bmatrix} \beta_i \\ 0 \end{bmatrix} f_i(p_{i-1})$$



$a_i, b_i > 0$: Degradation rates (1/Time constants)

$c_i, \beta_i > 0$: Production rates



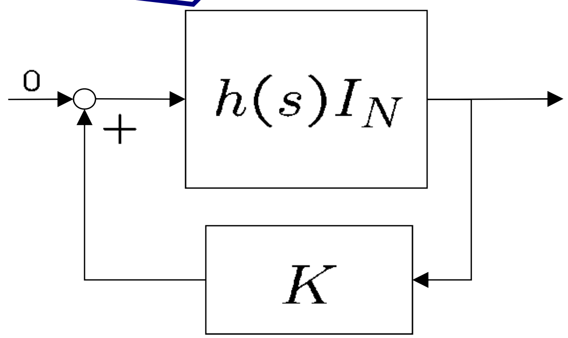
$f_i(p_{i-1})$: Hill function

$$f_i(p_{i-1}) := \begin{cases} \dfrac{p_{i-1}^{\nu}}{1 + p_{i-1}^{\nu}} & \text{(Mono. increasing for activation)} \\[2ex] \dfrac{1}{1 + p_{i-1}^{\nu}} & \text{(Mono. decreasing for repression)} \end{cases}$$

Linearized Gene Network Model

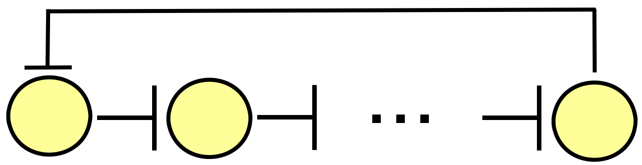
Each gene's dynamics

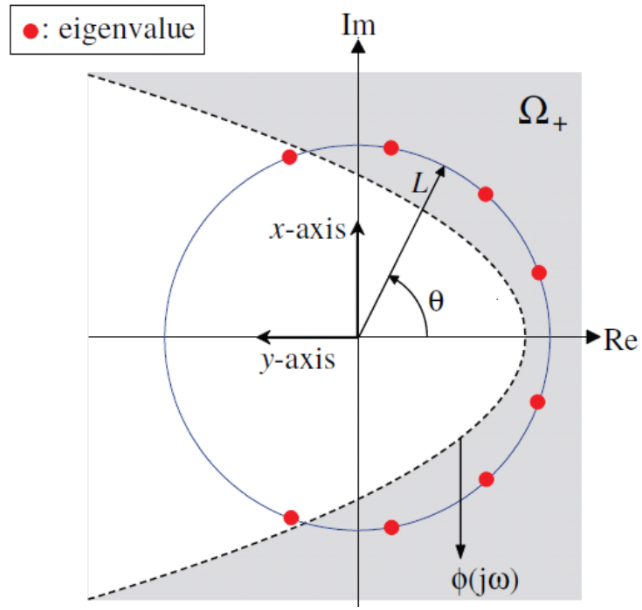
$$h(s) := \frac{1}{(T_a s + 1)(T_b s + 1)}$$

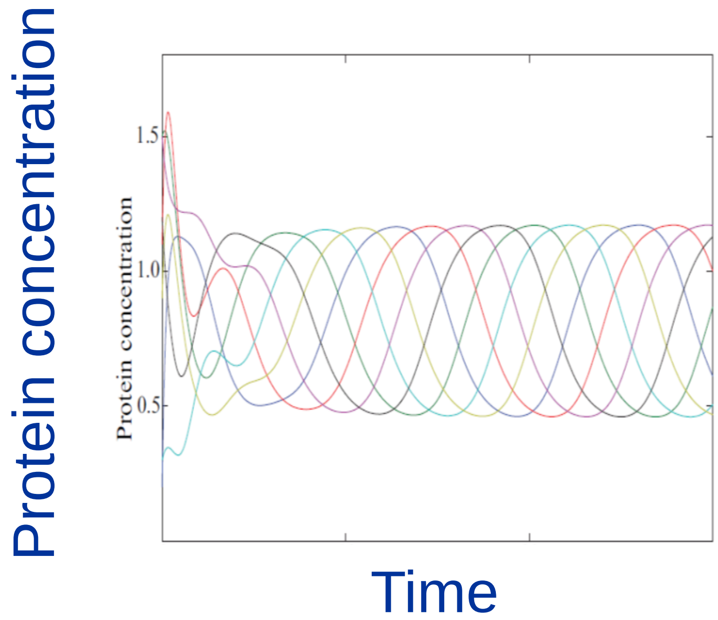


$$\begin{bmatrix} 0 & 0 & 0 & \cdots & R_1^2 f_1'(p_N^*) \\ R_2^2 f_2'(p_1^*) & 0 & 0 & \cdots & 0 \\ 0 & R_3^2 f_3'(p_2^*) & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & \cdots & R_N^2 f_N'(p_{N-1}^*) & 0 \end{bmatrix}$$

Interaction structure

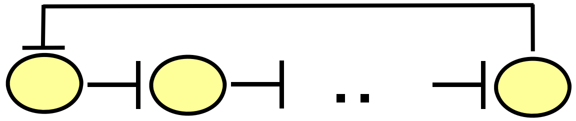






Analytic Criteria

- **Assumptions:**
  - All interactions are repressive



$R = \frac{\sqrt{c\beta}}{\sqrt{ab}}$

$\frac{\sqrt{T_a T_b}}{(\ldots T_b)/2}$

Theorem [Hori et al., 09]

The cyclic G...

a...

whe...

$\ldots(\pi/N) - \cos(\pi/N)$

$1 - \cos(\pi/N)$

$\ldots\cos(\frac{\pi}{N}))$

$\ldots(N, Q) + \nu Q(1 + \cos(\frac{\pi}{N}))$

Time-delay case

Non-homogeneous case

Robustness analysis

Experiments with RIKEN

- Four ...ntial parameters (N, ν, R, Q) which determine the existence of periodic oscillations
- This coincides with [H. E. Samad *et al.*, 05] N=3 , $Q = 1$

Robust Stability Condition

$$h(s) = \frac{1}{(T_a s + 1)(T_b s + 1)}$$

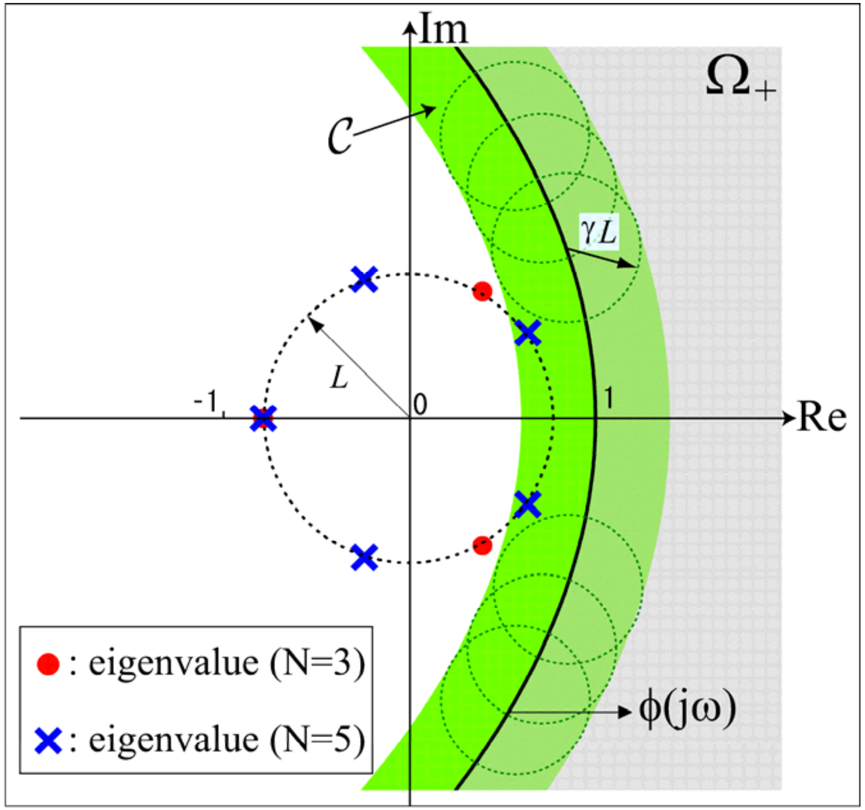
$$A = R^2 \begin{bmatrix} 0 & 0 & 0 & \cdots & \kappa_1 \\ \kappa_2 & 0 & 0 & \cdots & 0 \\ 0 & \kappa_3 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & \cdots & \kappa_N & 0 \end{bmatrix}$$

$$\left\| \frac{h}{1 - \lambda h} \right\|_\infty < \gamma, \quad \forall \lambda \in \sigma(A)$$

$^\exists D$: diagonal s.t.
DAD^{-1} is normal

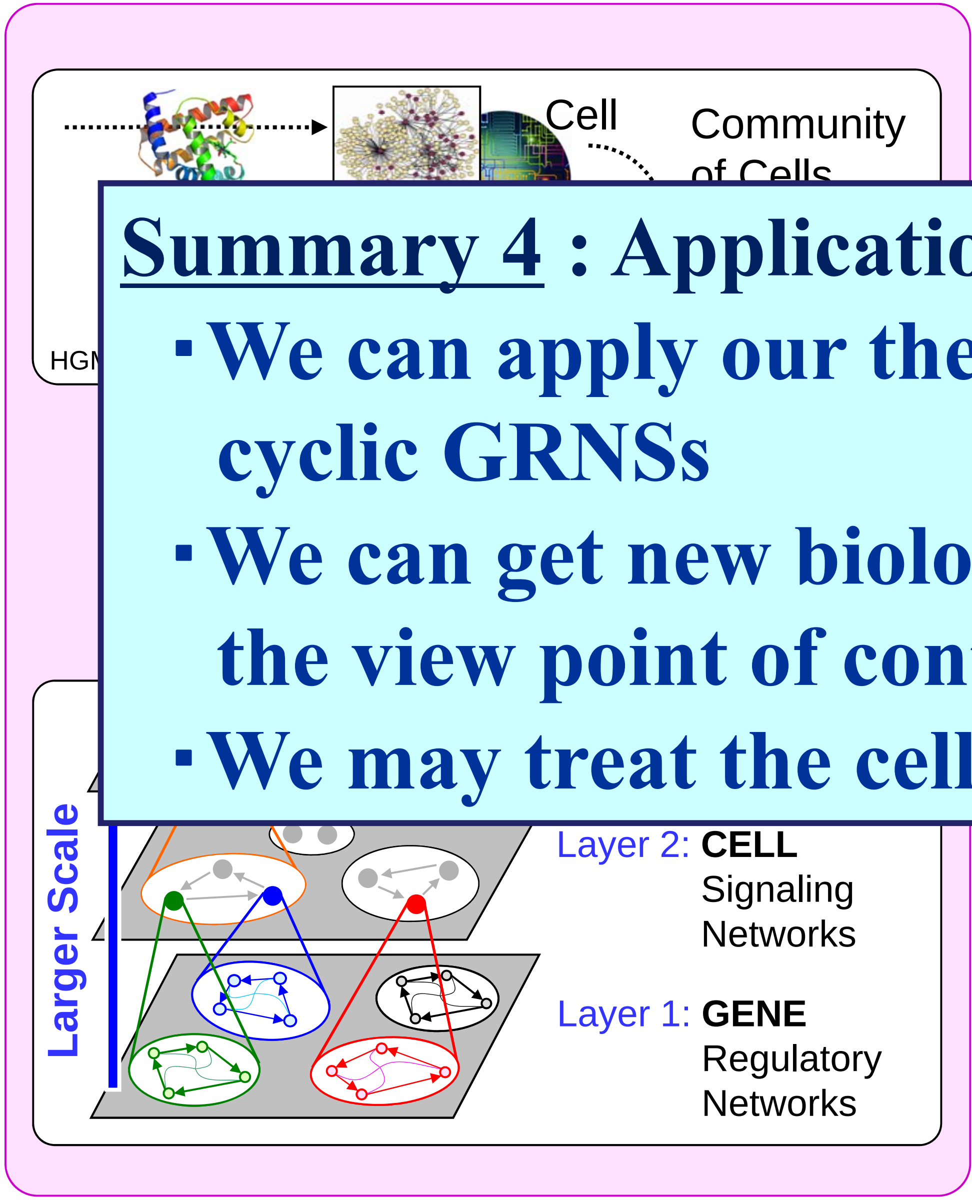
$$Q := \frac{\sqrt{T_a T_b}}{(T_a + T_b)/2} \quad R := \frac{\sqrt{c\beta}}{\sqrt{ab}}$$

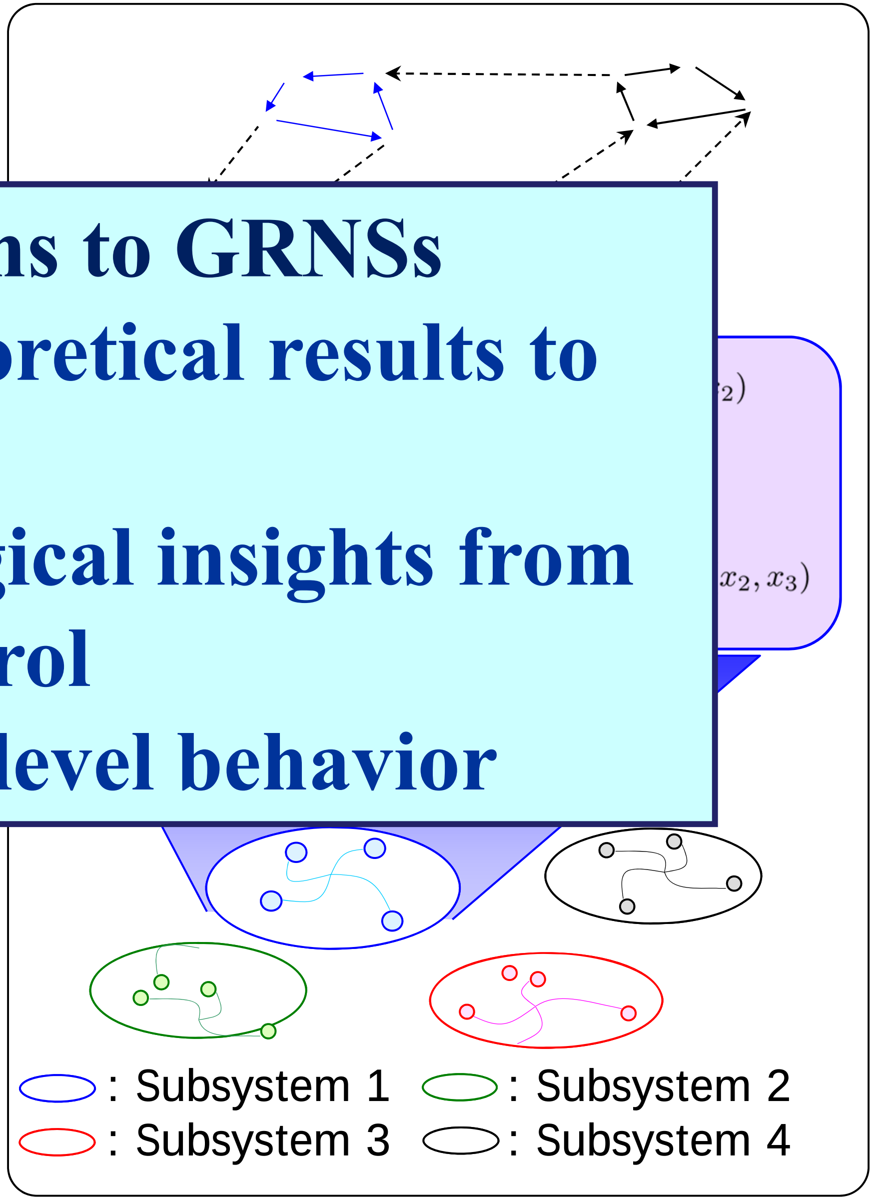
$$L := \prod_{k=1}^{N} \left| \frac{df_i}{dp}\Big|_{p^*} \right|^{\frac{1}{N}}$$



More Robust as N, R^2, Q, L decrease.

Hierarchical Bio-Network Systems





Summary 4 : Applications to GRNSs

- We can apply our theoretical results to cyclic GRNSs
- We can get new biological insights from the view point of control
- We may treat the cell level behavior

OUTLINE

1. Stability Analysis: Review
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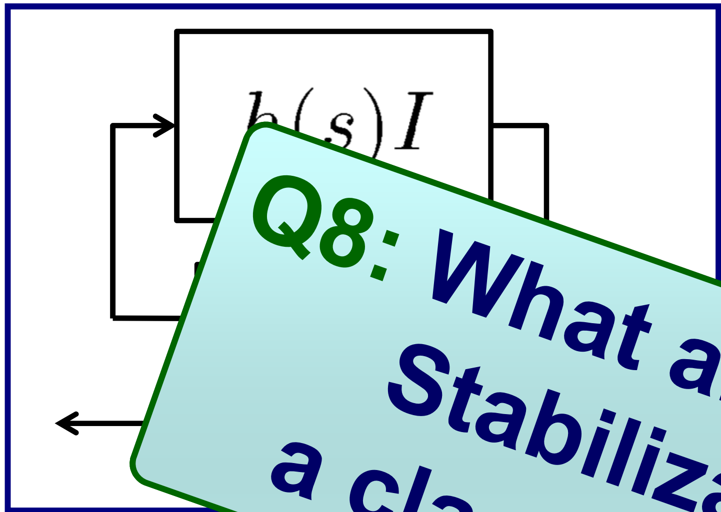
6. Concluding Remarks

Linear → Nonlinear



Q8: What are Robust Stability & Stabilizability Conditions for a class of Nonlinear Systems ?

Homogeneous
→ Heterogeneous

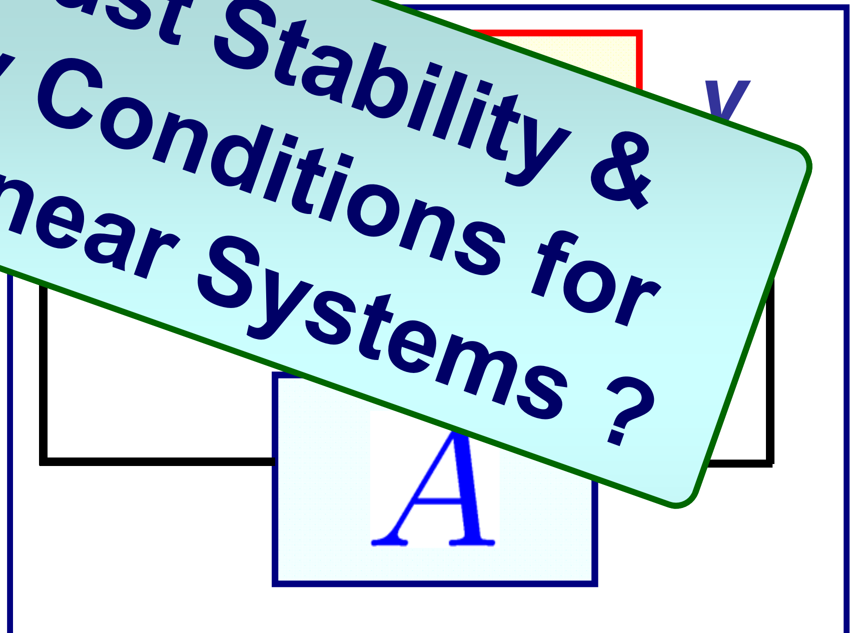
Robust Stability Analysis

$$\Sigma_i \; : \; h_i(s)$$

Linear → Nonlinear

Nonlinear Analysis

$$\Sigma_i \; : \; \mathcal{N}_i$$



Interconnection Matrix

(Q, S, R) Dissipativity

Definition

A system is called *(Q, S, R)-dissipative* if there exists a positive definite function $V(x)$ called *storage function*, such that for all $x \in \mathcal{X}$

$$V(x(T)) - V(x(0)) \leq \int_0^T w(u(t), y(t))\, dt$$

holds for all inputs $u \in \mathcal{U}$ and all finite $T \geq 0$, where $w(u, y)$ is quadratic supply rate given by

$$w(u, y) = y^T Q y + 2 y^T S u + u^T R u$$

with $R = R^T \in \mathbb{R}^{m \times m}$, $S \in \mathbb{R}^{p \times m}$, $Q = Q^T \in \mathbb{R}^{p \times p}$.

Stability for Dissipative Agents

(Hirsch, Hara: IFAC2008)

Agent Dynamics — SISO (Q, S, R)-dissipative

$$\begin{aligned} \dot{x}_i &= f_i(x_i) + g_i(x_i)u_i \\ y_i &= h_i(x_i) \end{aligned}$$

$Q = \text{diag}\{Q_i\} \leq 0$,
$S = \text{diag}\{S_i\}$,
$R = \text{diag}\{R_i\} \geq 0$.

Theorem (LMI)

$V := \sum_{i=1}^{N} d_i \cdot V_i$

If $\exists$ a diagonal matrix $D > 0$ such that

$$A^T DRA + DSA + A^T S^T D + DQ < 0$$

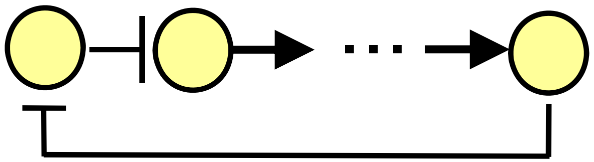
holds, then the networked system is asymptotically stable.

If $R = 0$ and $S > 0$, then
$A + S^{-1}Q/2$: diagonally stable

Stability Condition for GRNs

Cyclic Structure with Negative Feedback

$$A = \begin{bmatrix} 0 & 0 & 0 & \cdots & -1 \\ 1 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 & 0 \end{bmatrix}$$



$$A + S^{-1}Q/2$$

$$= \begin{bmatrix} -1/\gamma_1 & 0 & 0 & \cdots & -1 \\ 1 & -1/\gamma_2 & 0 & \cdots & 0 \\ 0 & 1 & -1/\gamma_3 & \cdots & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 & -1/\gamma_N \end{bmatrix}$$

$$\mathrm{diag}\{1/\gamma_i\} \begin{bmatrix} -1 & 0 & 0 & \cdots & -\gamma_1 \\ \gamma_2 & -1 & 0 & \cdots & 0 \\ 0 & \gamma_3 & -1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & \cdots & \gamma_N & -1 \end{bmatrix}$$

Secant Criterion

$$\gamma_1 \cdots \gamma_N < \sec(\pi/N)^N$$

Diagonally Stable

(Arcak & Sontag. Automatica, 2006)

Stabilization

Theorem (LMI)

If there exist a solution (X, Y) ... ep-
resented by

$Y > 0,$ ...

XY^{-1}

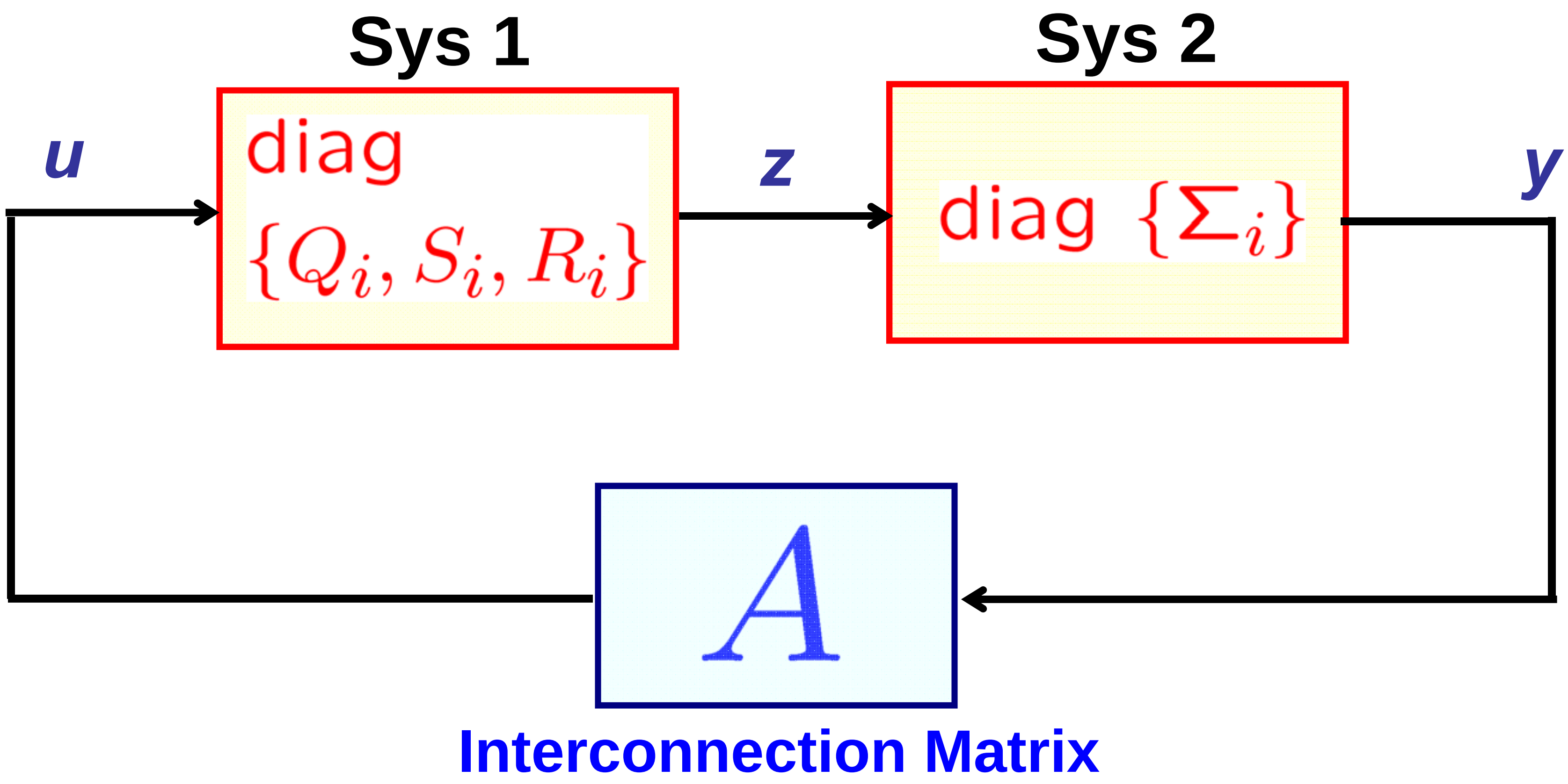
... networked system.

Q9 : Dissipative (Passive) → beyond Dissipative (Passive) ?
Can we apply the results to non-dissipative systems ?

Note: *X* preserves the network structure of *A*,
since *Y* is diagonal.

MADS with Cascaded Dissipative Systems

A class of multi-agent dynamical systems
based on dissipative properties



Two Cascaded Dissipative Systems

Sys 2

$$\text{diag}\ \{Q_i, S_i, R_i\} \Leftrightarrow A = \begin{bmatrix} 0 & I_N \\ A_2 & 0 \end{bmatrix} \in \mathbb{R}^{2N \times 2N}$$

$$Q = \text{diag}\{Q_1, Q_2\},$$
$$S = \text{diag}\{S_1, S_2\},$$
$$R = \text{diag}\{R_1, R_2\}$$

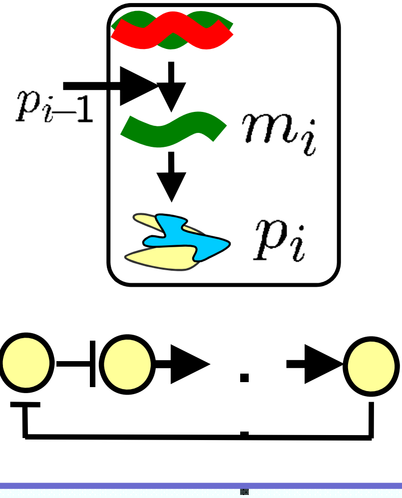
Stability Condition (LMI)

$$\begin{bmatrix} D_1 Q_1 & D_1 S_1 + A_2^T S_2 D_2 & 0 & A_2^T R_2 D_2 \\ D_1 S_1 + D_2 S_2 A_2 & D_2 Q_2 & D_1 R_1 & 0 \\ 0 & D_1 R_1 & -D_1 R_1 & 0 \\ D_2 R_2 A_2 & 0 & 0 & -D_2 R_2 \end{bmatrix} < 0$$

$D_1 > 0,\ D_2 > 0$: diagonal

Gene Regulatory Network

$$Q = \mathrm{diag}\{Q_1, Q_2\},$$
$$S = \mathrm{diag}\{S_1, S_2\},\ R = 0$$



Stability Condition (LMI)

$$\begin{bmatrix} D_1 Q_1 & D_1 S_1 + A_2^T D_2 S_2 \\ S_1 D_1 + S_2 D_2 A_2 & D_2 Q_2 \end{bmatrix} < 0$$

Stabilization Condition (LMI)

$$\begin{bmatrix} Y_1 Q_1 & Y_1 S_1 + X_2{}^T S_2 \\ S_1 Y_1 + S_2 X_2 & Y_2 Q_2 \end{bmatrix} < 0 \qquad A_2 = X_2 Y_2{}^{-1}$$

Dissipative + Integrator

Sys 2

$\frac{1}{}\cdot I$

Theorem

Suppose $R = 0$ and $Q < 0$.

Summary 5 : Dissipative Networked Systems

- **LMI stability & stabilizability conditions**
- **Stability and robust stability conditions**
- ***D*-scaling technique is also useful**
- **Dissipative → Non-dissipative**

P

$^{\exists}D$: positive diagonal s.t.

$A_{2D} := DA_2D^{-1}$ is normal. Then,

A_2 is stable $\Leftrightarrow$ A_2 is diagonally stable .

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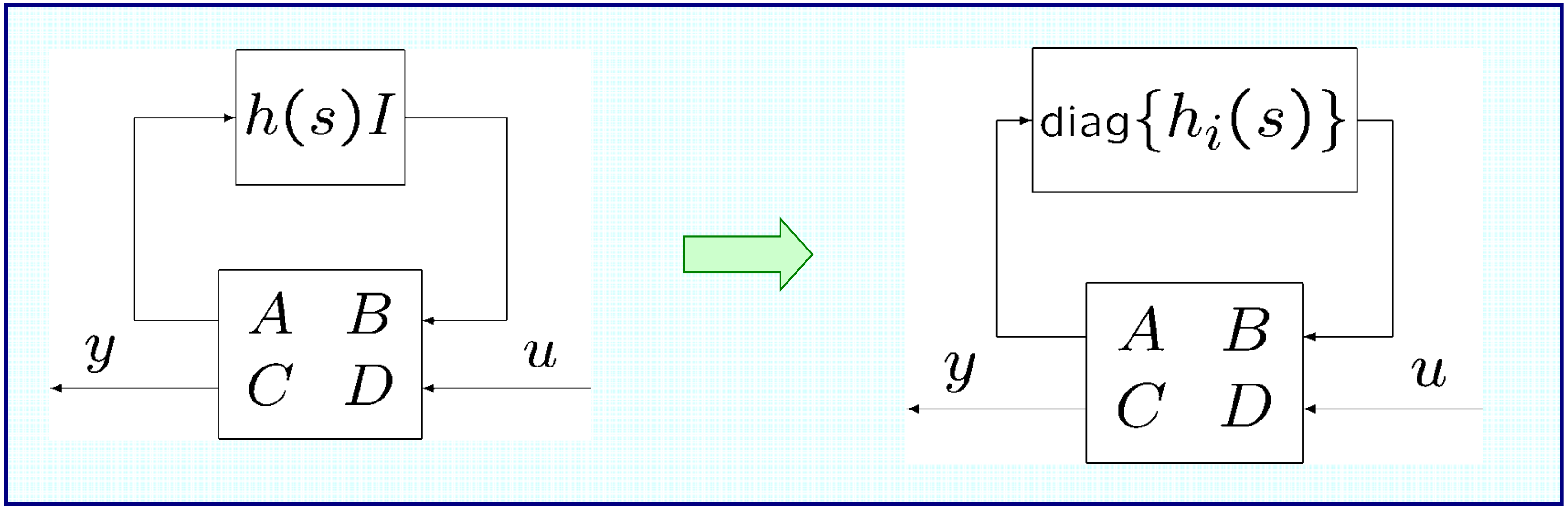
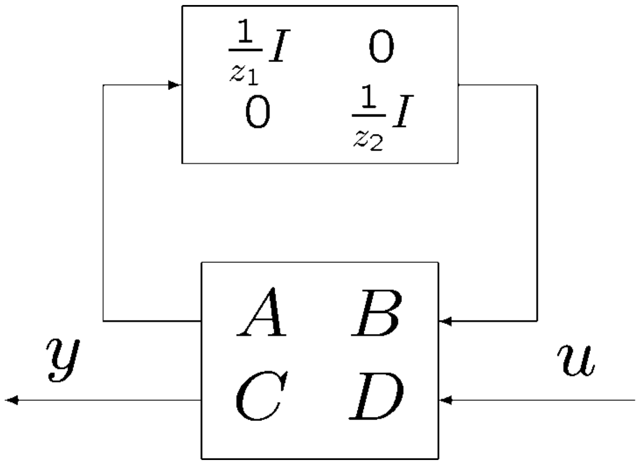
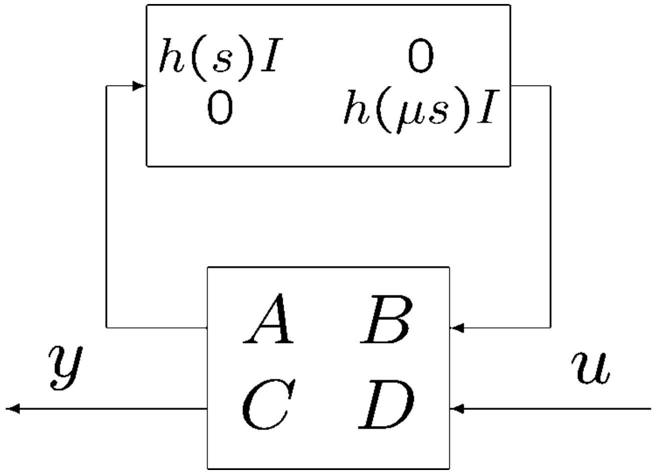
New Framework for System Theory





2D System

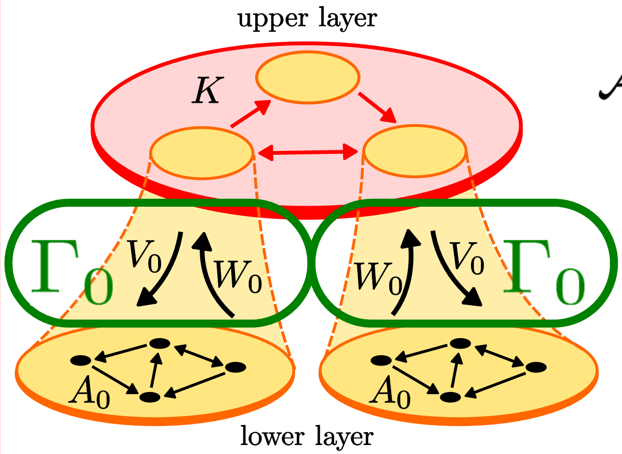


Singular Perturbed System

Multi-resolved Systems

Homogeneous vs Heterogeneous

Homogeneous

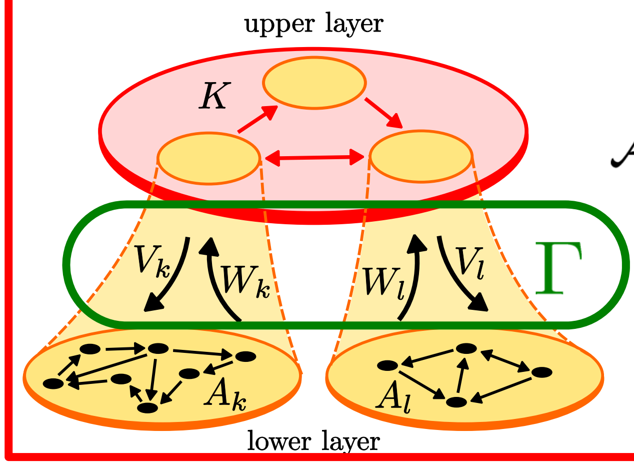


$$\mathcal{A} = \begin{bmatrix} A_0 & & \\ & \ddots & \\ & & A_0 \end{bmatrix} + \begin{bmatrix} k_{11}\Gamma_0 & \cdots & k_{1M}\Gamma_0 \\ \vdots & \ddots & \vdots \\ k_{M1}\Gamma_0 & \cdots & k_{MM}\Gamma_0 \end{bmatrix}$$

$$= I_M \otimes A_0 + K \otimes \Gamma_0 \ ; \ \Gamma_0 = V_0 W_0^\top$$

$\mathcal{A}$ can be written using Kronecker $\bigotimes$ product .

Heterogeneous



$$\mathcal{A} = \begin{bmatrix} A_1 & & \\ & \ddots & \\ & & A_M \end{bmatrix} + \begin{bmatrix} k_{11}\Gamma_{11} & \cdots & k_{1M}\Gamma_{1M} \\ \vdots & \ddots & \vdots \\ k_{M1}\Gamma_{M1} & \cdots & k_{MM}\Gamma_{MM} \end{bmatrix}$$

$$= \mathrm{diag}\{A_k\} + K \odot \Gamma$$

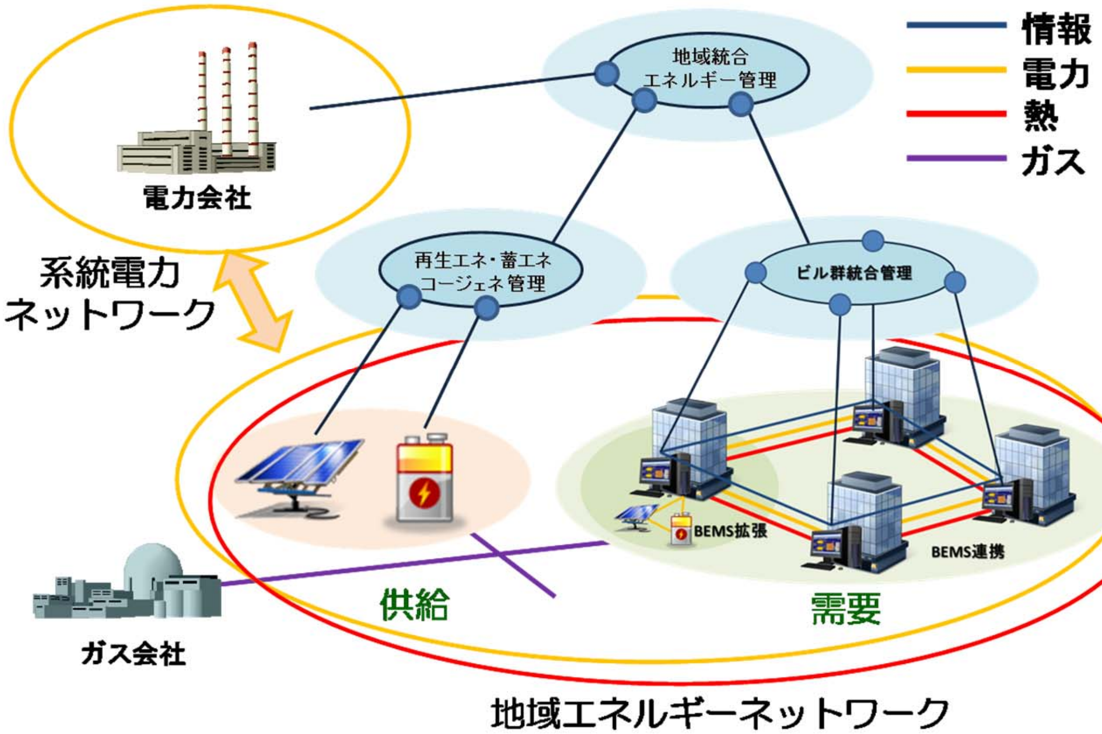
We need Khatori-Rao product $\odot$.

Smart Energy NW and Energy Saving

Smart Energy Network

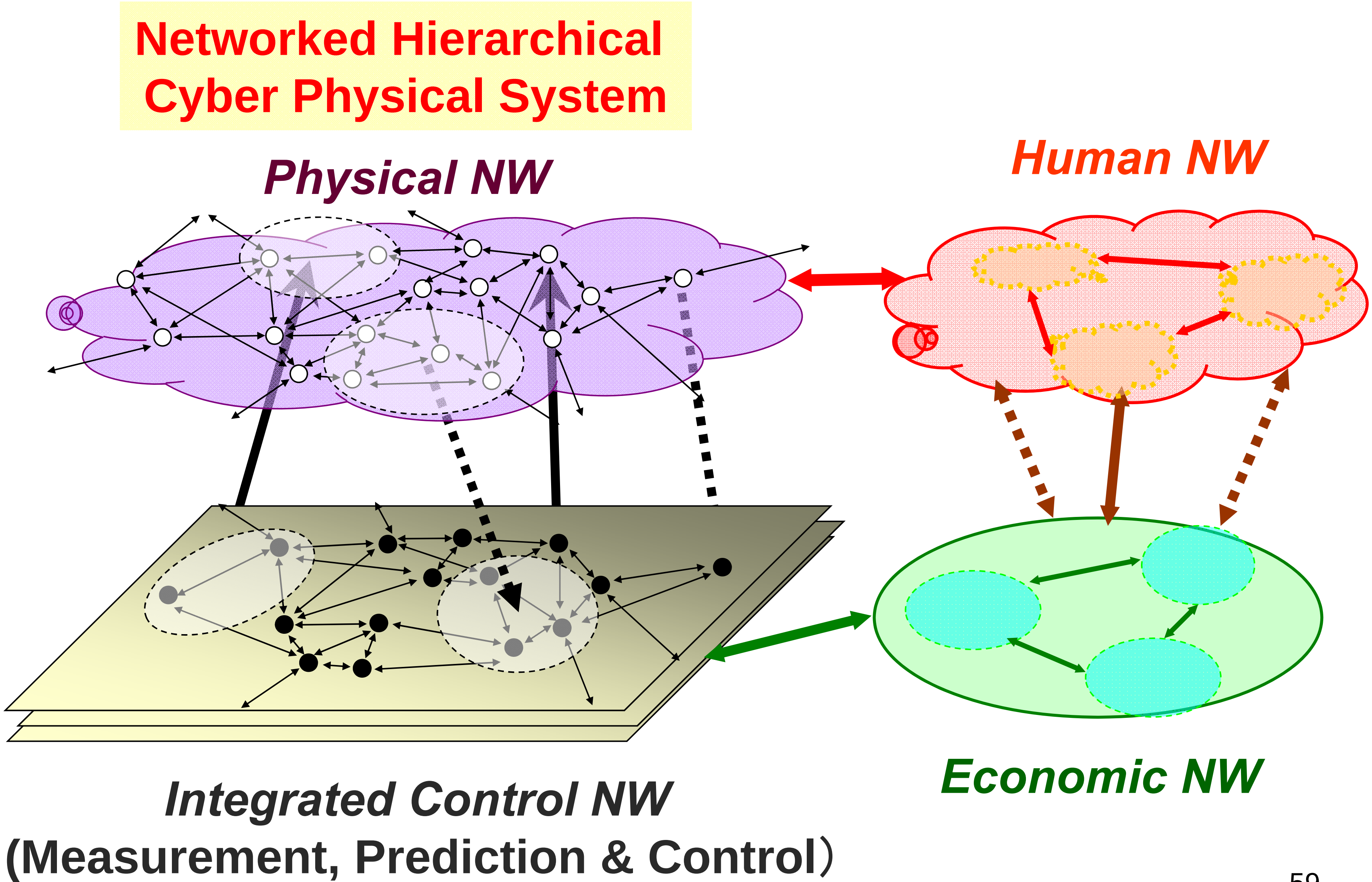
Electric power network
+ Gas energy network

Multi-resolved Hierarchical Modeling
→ Multi-resolved Prediction
→ Hierarchical Decentralized Control



U Tokyo
Tokyo Gas
Fujitsu
Azbil

Harmony with Nature and Social System



Acknowledgements

① Glocal Control

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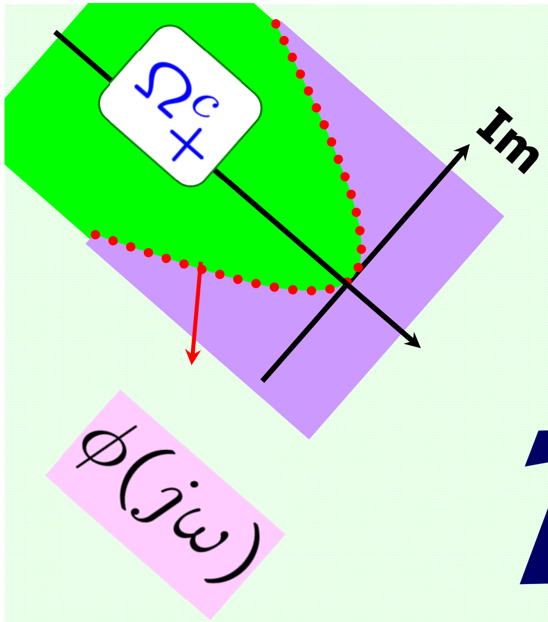
④ Nonlinear Analysis

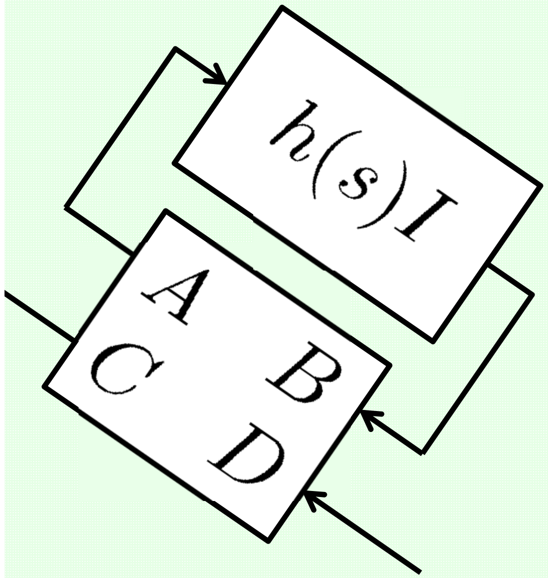
Sandra Hirsch (TU Munich)

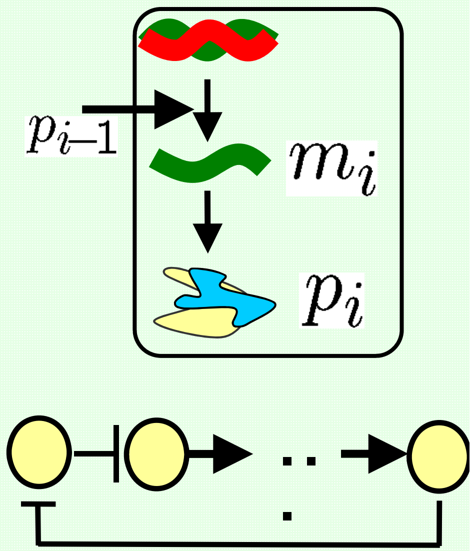


(ii) $\left\| \frac{h}{1-\lambda h} \right\|_\infty < \gamma, \ \forall \lambda \in \sigma($

(iii) $\left| \frac{1}{\phi - \lambda} \right| < \gamma, \ \forall \lambda \in \sigma(M),$

Thank you very much !





Framework for Glocal Control

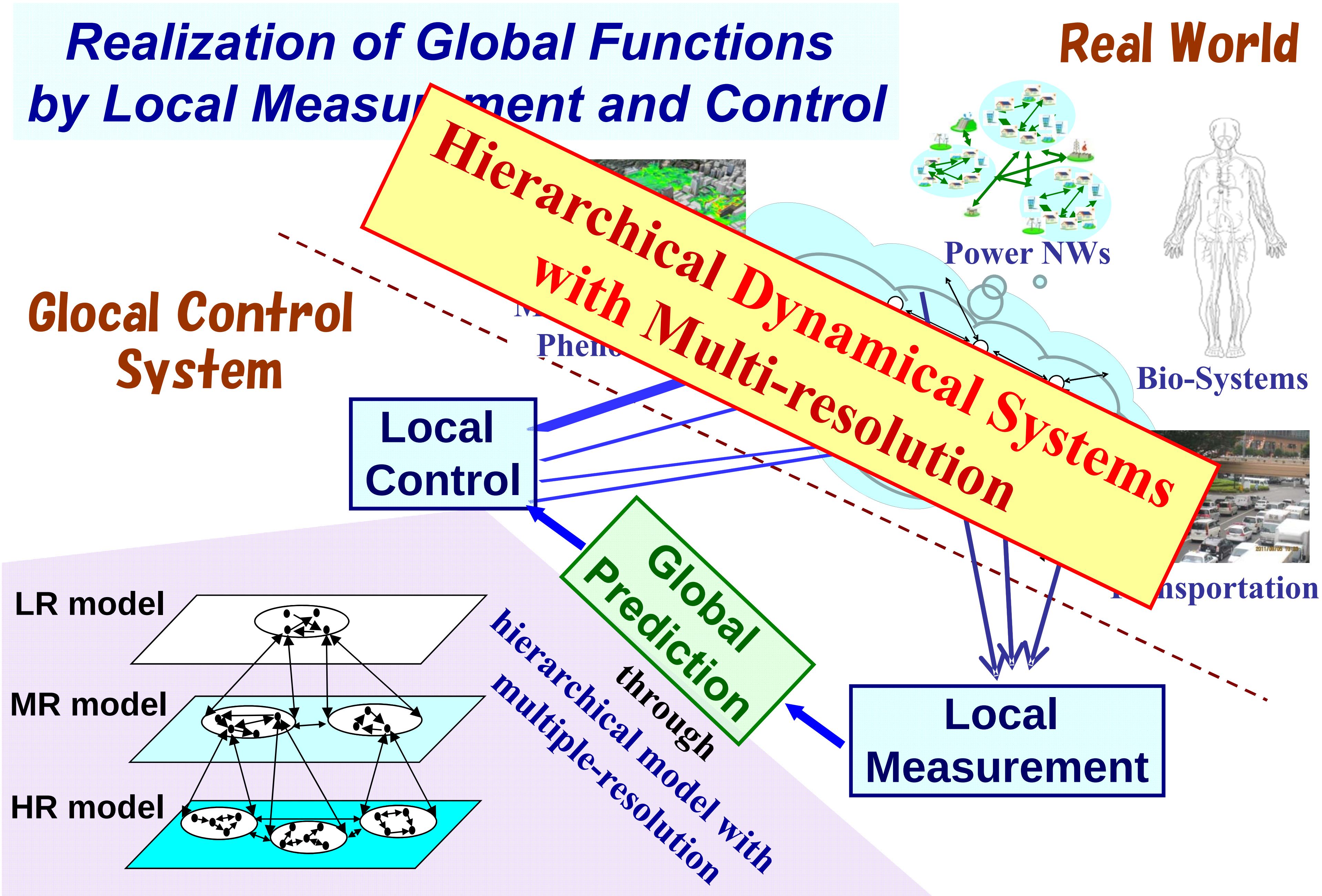


Image of Glocal Control System