

**Workshop: Toward Glocal Control**  
The University of Tokyo, 2010.4.1

Idea of  
“Glocal Control”

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# OUTLINE

1. What is “Glocal Control” ?
2. Global vs Local
3. Hierarchical Consensus
4. Toward “Glocal Control”

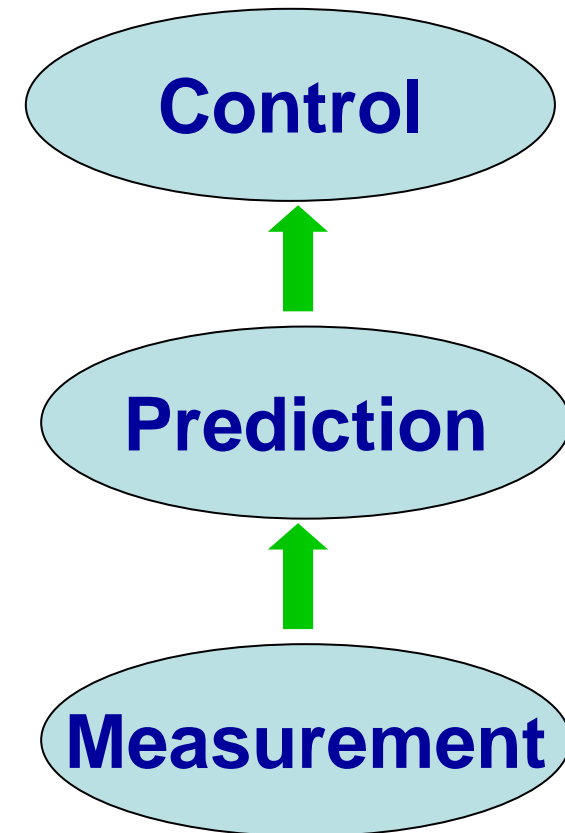
# Glocal Control

## Realization of Global Functions by Local Measurement and Control

Recently, systems to be treated in various fields of engineering including control have become large and complex, and more high level control such as adaptation against changes of environments for open systems is required. We need to pay much attention to analysis and control of meteorological phenomena and bio systems as such large scale dynamical systems, where our available actions of measurement and control are restricted locally although our main purpose is to achieve the desired global behaviors.

This motivates us to develop a **new research area** so called "**Glocal Control**," which means that the **desired global behaviors is achieved by only local actions.**

# Urban Heat Island Problem



**Glocal Control**

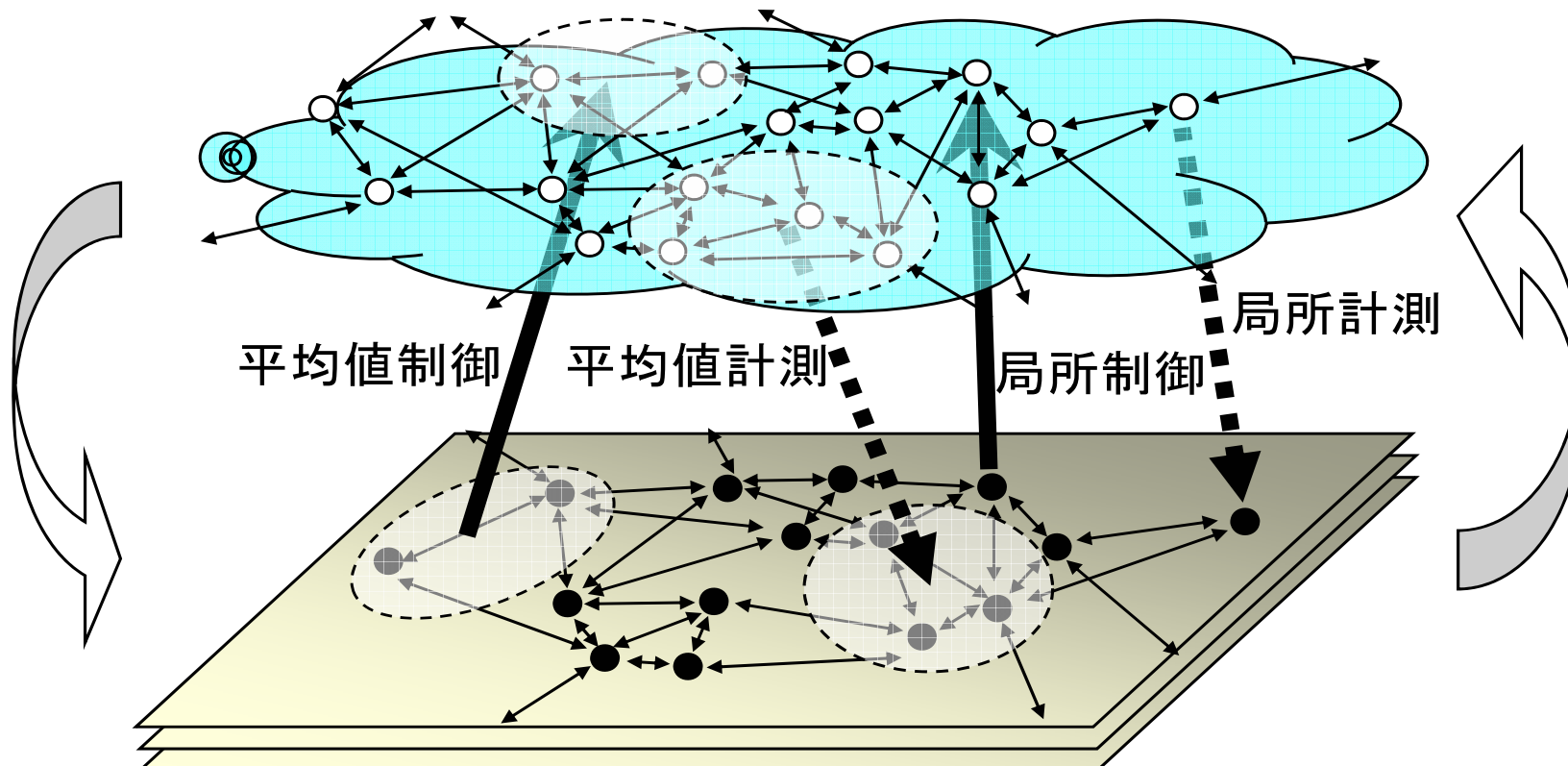
グローバル制御



☆ : Fans      ○ : Temperature, Wind Sensors

# Framework for "Glocal Control"

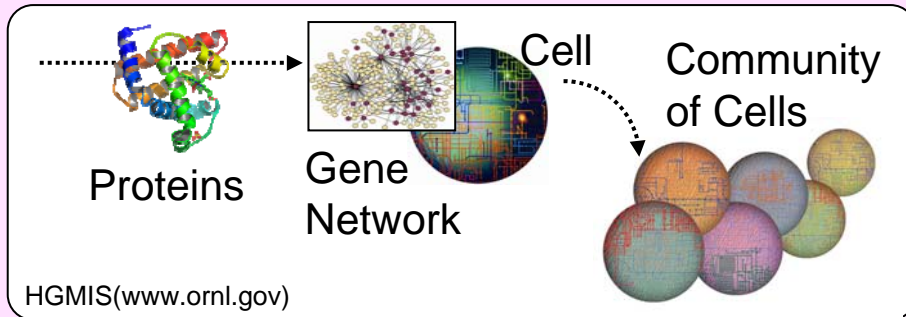
## Physical Network



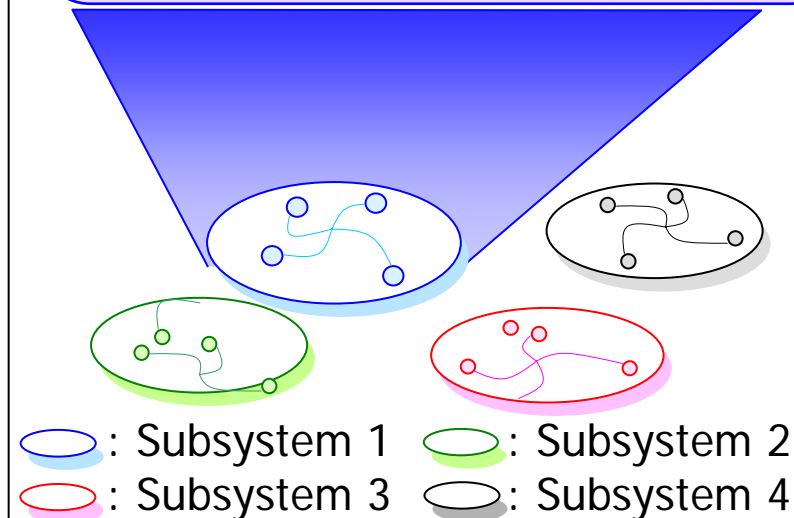
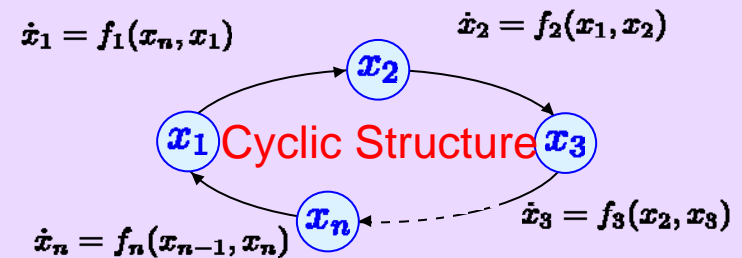
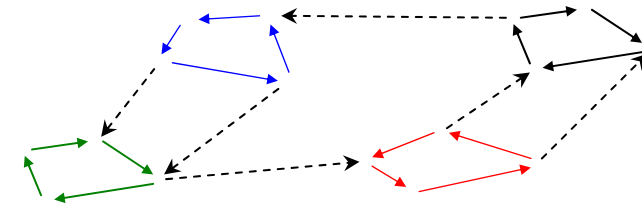
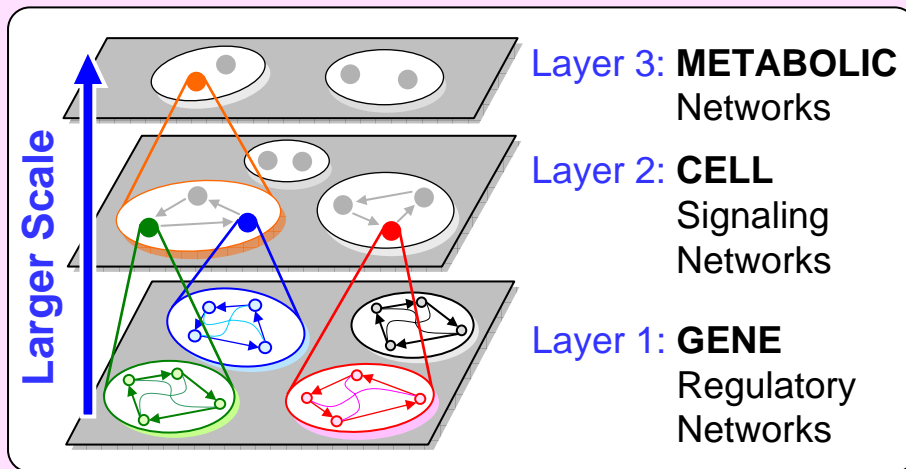
## Information (Logical) Network

Measurement → Prediction → Control

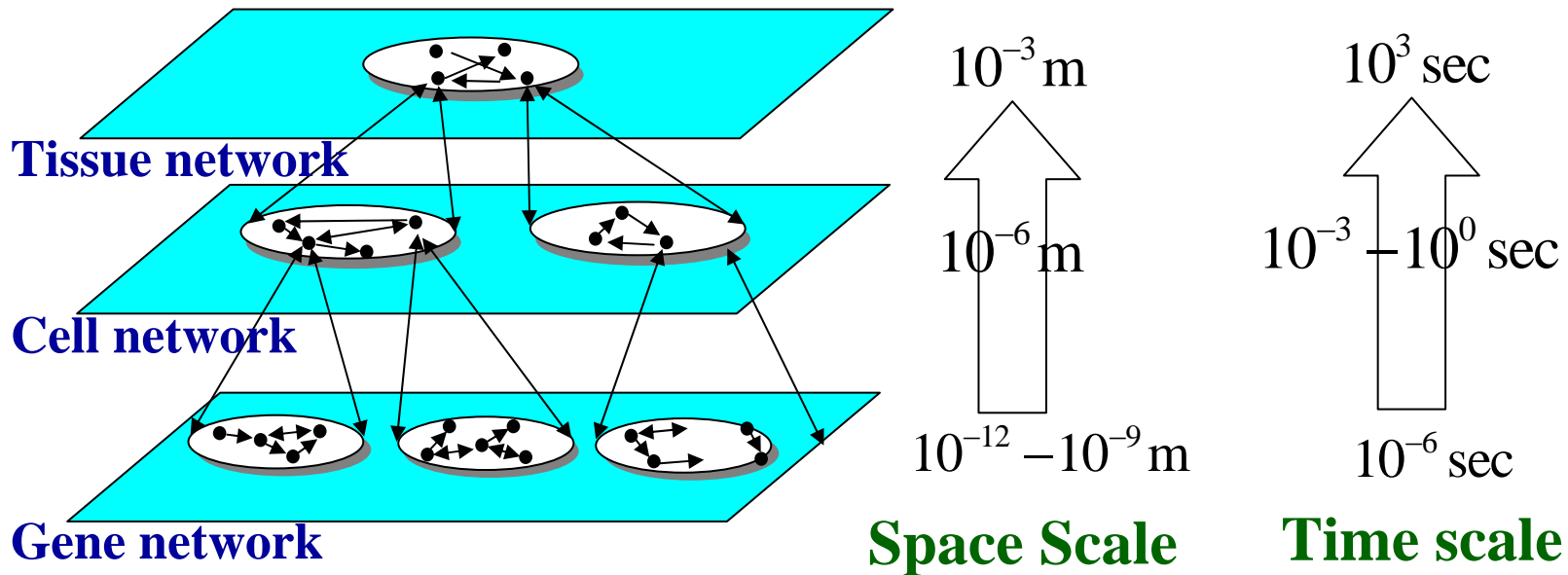
# Hierarchical Bio-Network Systems



## Hierarchical Bio-Network Systems



# Hierarchical Structure with Multi-resolution



- Resolution of State Variables
- Resolution Converters (High  $\Leftrightarrow$  Low)

# Key Notion Toward “Glocal Control”

- (1) Combined NW:  
Physical NW & Information NW**
- (2) Seamless Actions of  
Measurement, Prediction, Control**
- (3) Hierarchical Dynamical Systems  
with Multi-Resolution**



# OUTLINE

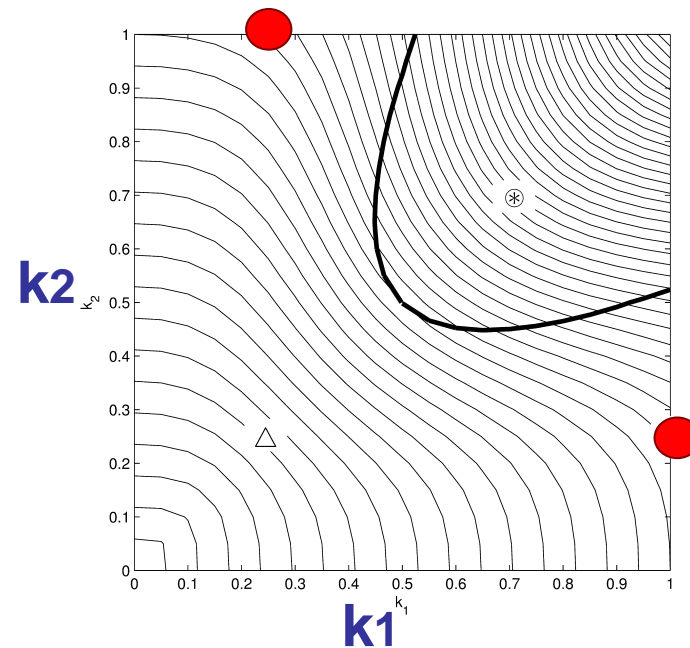
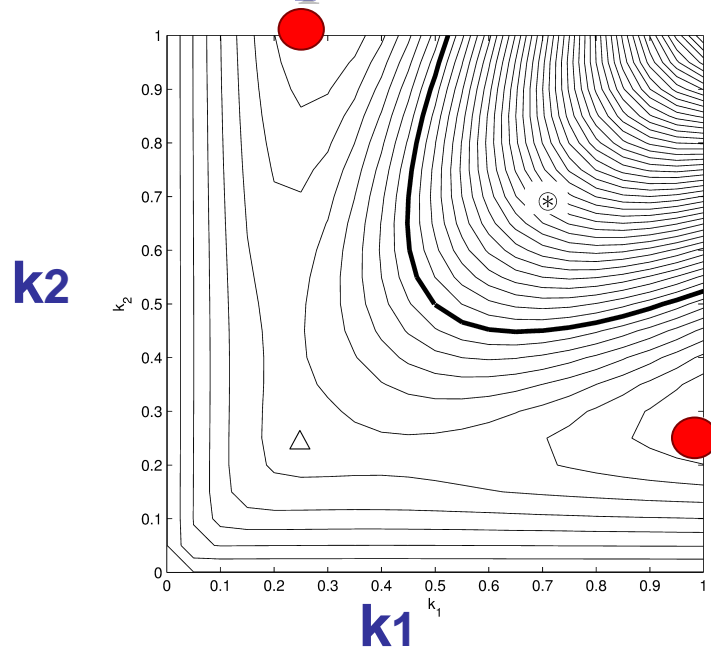
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# Example : Glocal vs Local Prisoners' Dilemma (1/2)

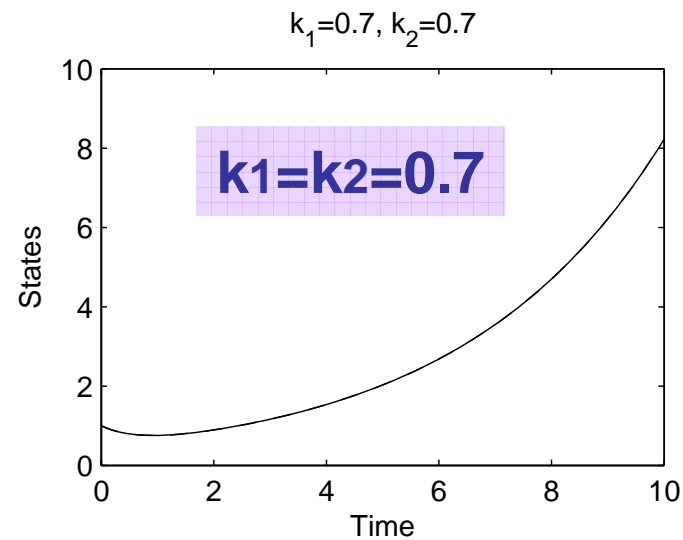
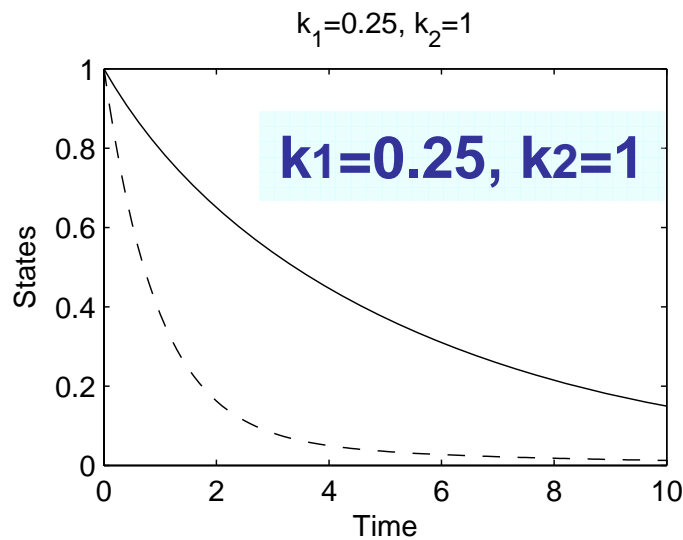
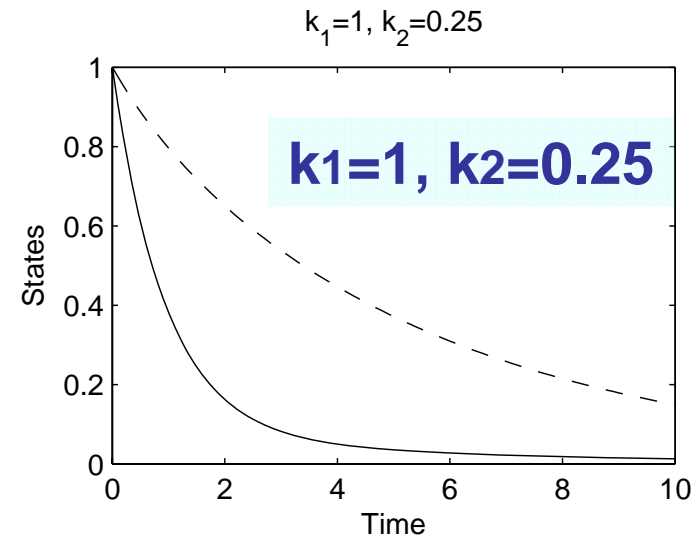
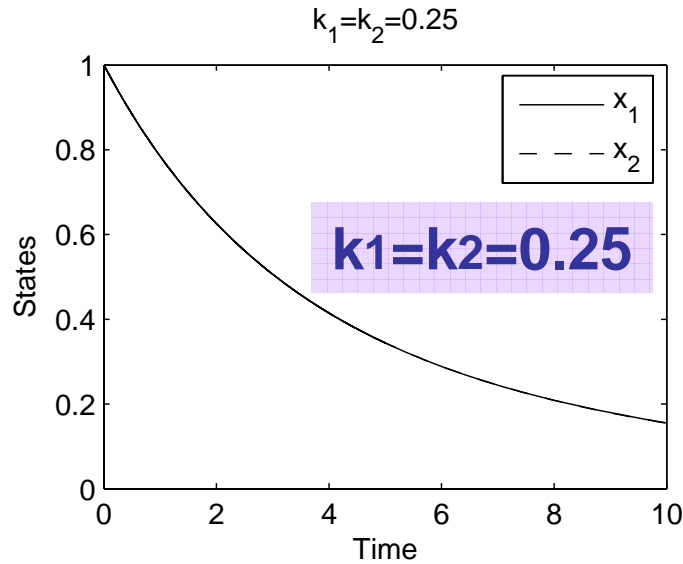
## System

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -k_1 & \frac{2k_2^2}{1+2(k_1-k_2)^2} \\ \frac{2k_1^2}{1+2(k_1-k_2)^2} & -k_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

## Closed-loop Poles



# Example : Glocal vs Local Prisoners' Dilemma (2/2)



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# Consensus for multi-agent dynamical systems

$$\dot{x}_i(t) = u_i(t) = \sum_{j \neq i} a_{ij} (x_j(t) - x_i(t))$$

$$\iff \dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) \quad \mathbf{A} : \text{Graph Laplacian}$$

$$\exists \xi, \quad \lim_{t \rightarrow \infty} \mathbf{x}(t) = \xi \cdot \mathbf{1}$$

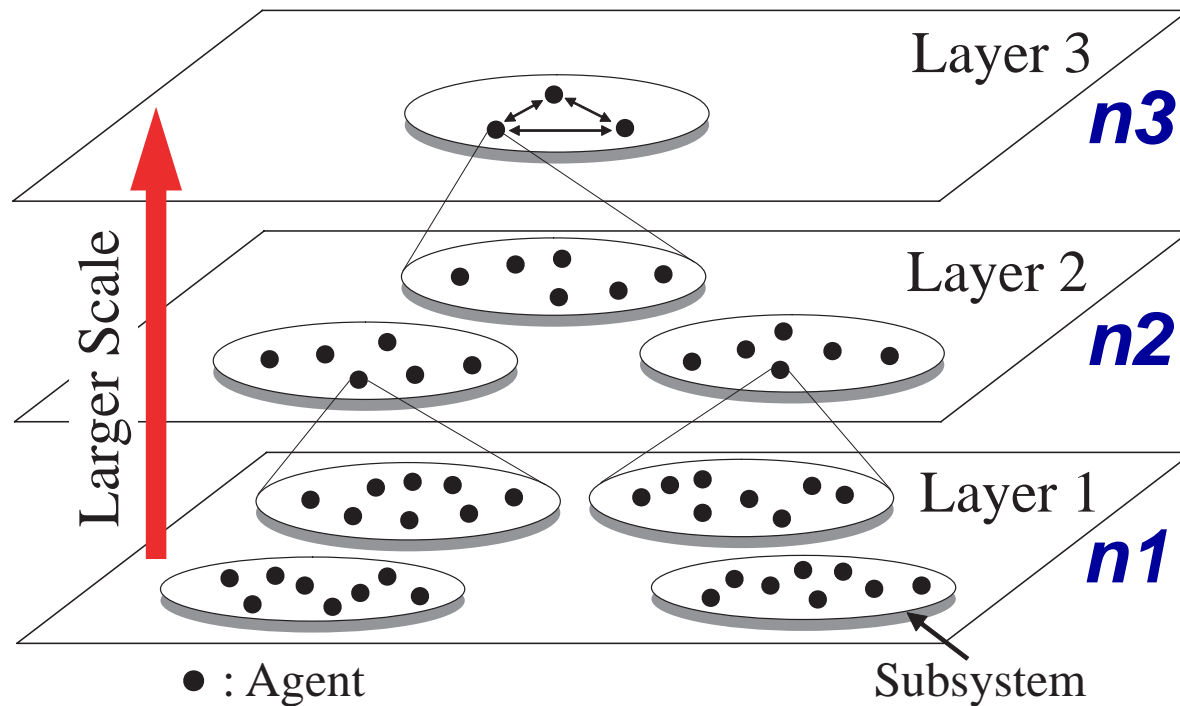
$$\mathbf{x} \equiv (x_1, \dots, x_n)^T$$

$$\mathbf{1} \equiv (1, \dots, 1)^T$$

## Fundamental Questions

- *Consensus = Lyapunov Stability of total system, which is quite large ?*
- *Consensus Performance = Stability Degree ?*

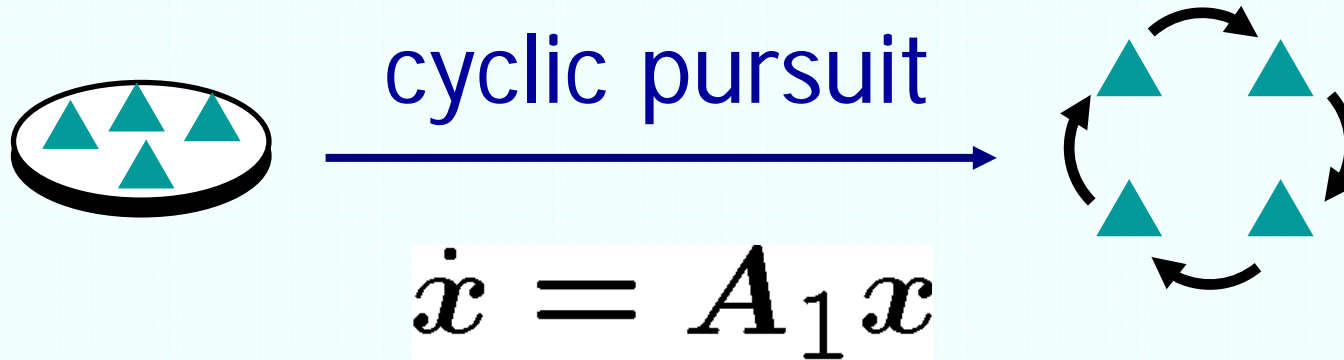
# Hierarchical Multi-agent Dynamical System



# total agents :  $n1 \times n2 \times n3$

• *Hierarchical model with fractal structure ?*

# Information Structure Inside Sub-group



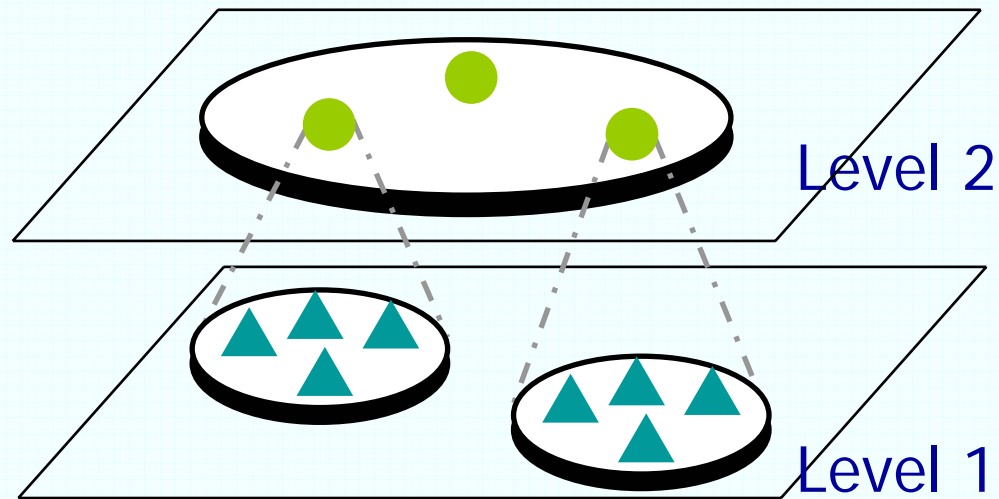
$$A_1 = P - I = \begin{pmatrix} -1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 1 \\ 1 & 0 & 0 & -1 \end{pmatrix}$$

Graph Laplacian

$$P = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{pmatrix}$$

Circulant matrix

# Hierarchical Information Structure (1/2)



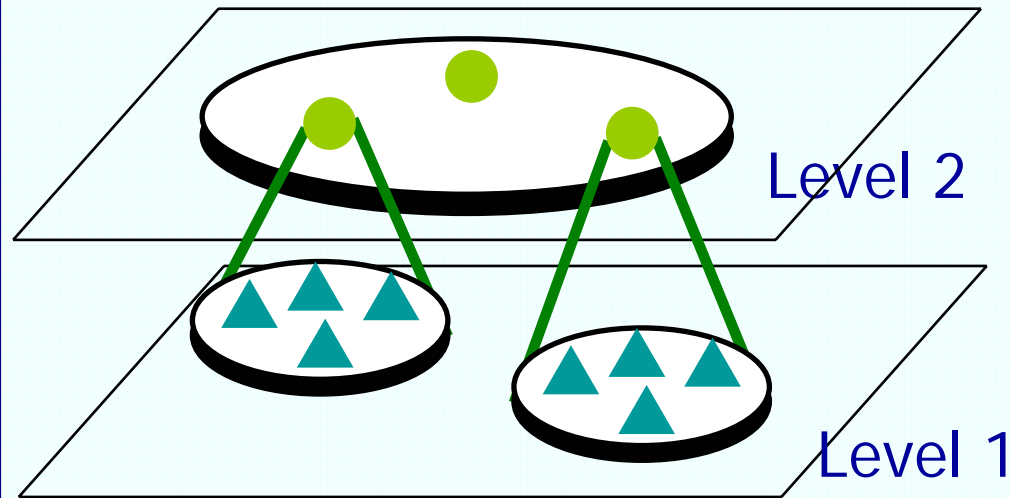
No interaction  
among  
sub-groups

$$\dot{x} = A_2 x$$

$$A_2 = \begin{pmatrix} A_1 & 0 & 0 \\ 0 & A_1 & 0 \\ 0 & 0 & A_1 \end{pmatrix}$$



# Hierarchical Information Structure (2/2)



Interaction  
with  
Fractal Structure

$$A_2 = \begin{pmatrix} A_1 - I & \Delta & 0 \\ 0 & A_1 - I & \Delta \\ \Delta & 0 & A_1 - I \end{pmatrix}$$

# General Expression for Hierarchical MADsS - Fractal & Cyclic Structure -

$A_1 = P - I$  : Cyclic Pursuit inside sub-group

$$A_l = \text{diag}(\underline{\underline{A_{l-1} - I}}) + \underline{\underline{P}} \otimes \underline{\underline{\Delta}}$$

Homogeneous  
structure

Fractal  
structure

**Property on  
Interactions**

weak interaction:

Sparse  
Small gain

# New Weak Interconnection Properties

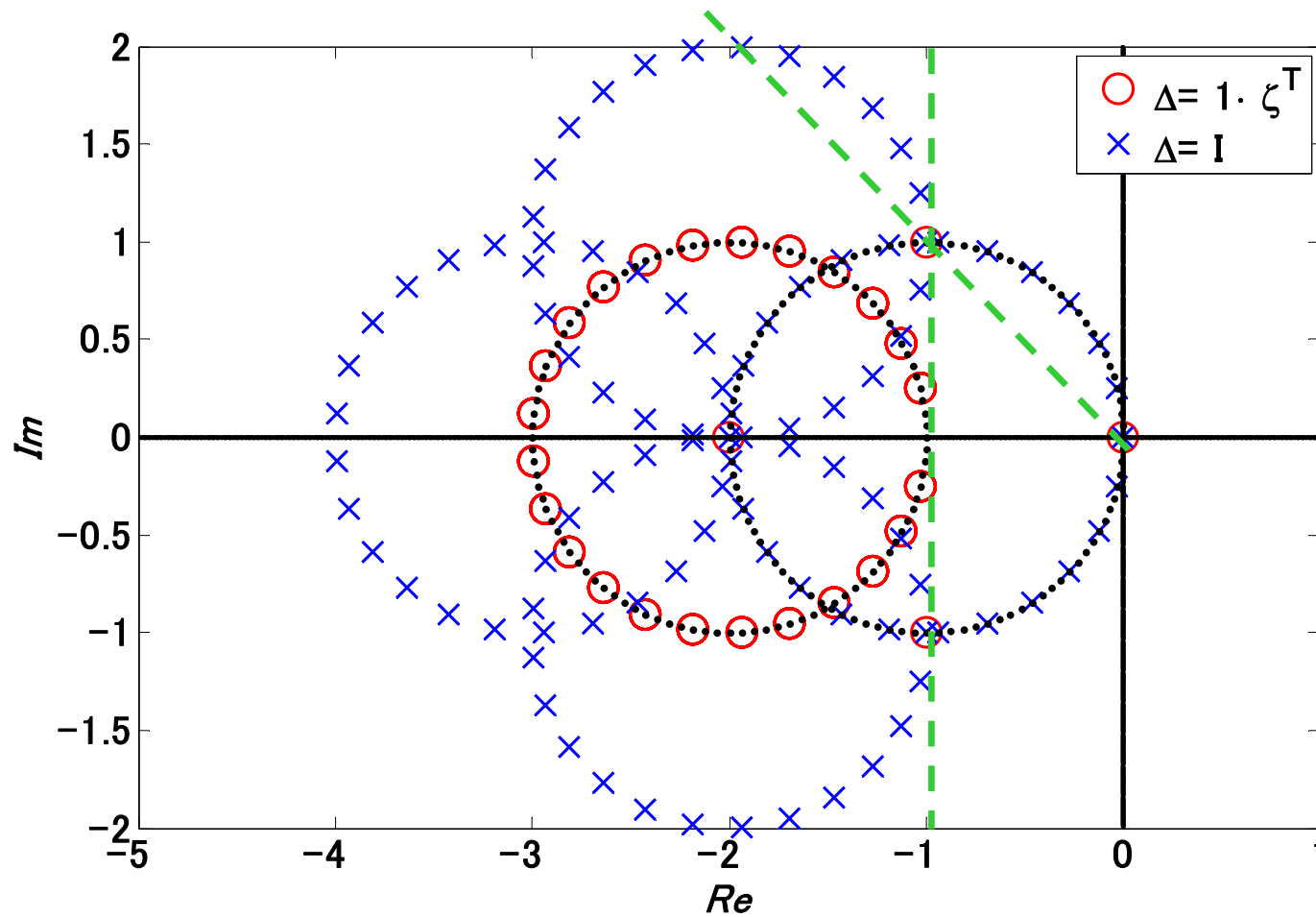
| $\Delta$ | Low-rank   | High-rank            |
|----------|--|----------------------|
| Sparse   | $\Delta = 1 \cdot \zeta^T$ $\left( \sum_{i=1}^n \zeta_i = 1, \zeta_i \geq 0 \right)$ | $\Delta = I$ (Smith) |
| Dense    |  |                      |

**Share an aggregated information  
Control uniformly**

$$\zeta^T = \begin{pmatrix} 1 & 0 & \dots & 0 \end{pmatrix} : \text{sparse}$$

$$\zeta^T = \frac{1}{n} \begin{pmatrix} 1 & 1 & \dots & 1 \end{pmatrix} : \text{dense}$$

# Eigenvalue Distributions ( $n_1=25, n_2=4$ )



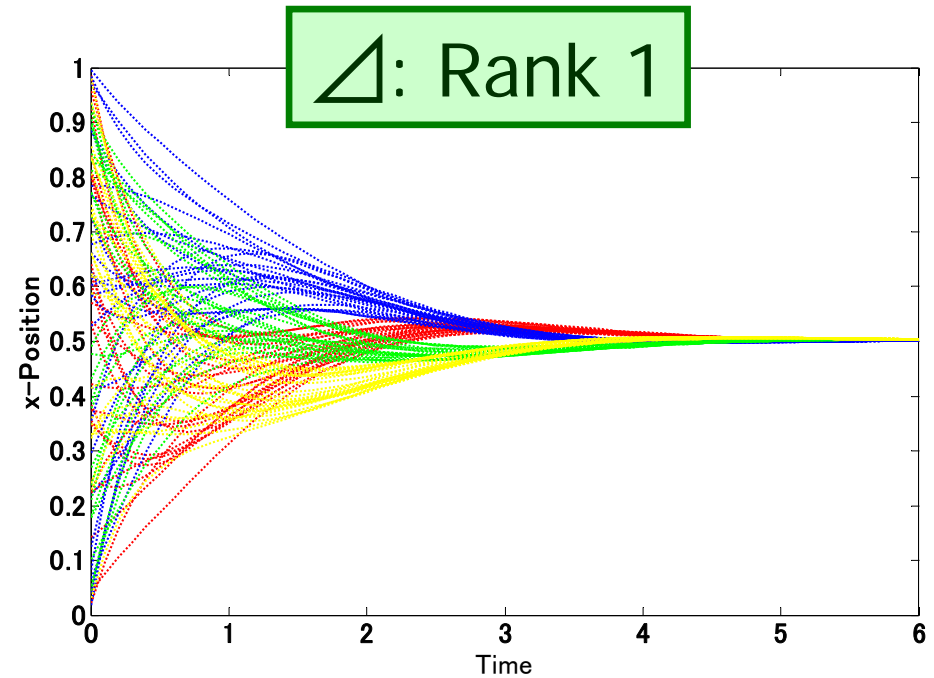
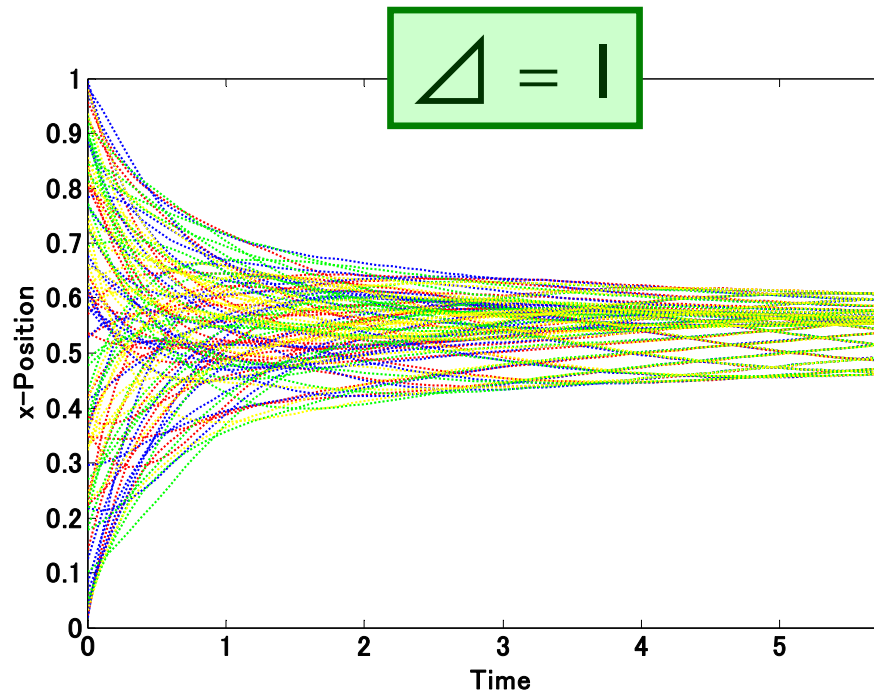
$n_1 > n_2$

○ : rank 1

× : Identity

# Time Responses ( $n_1=25, n_2=4$ )

$n_1 > n_2$



**Rapid Consensus**

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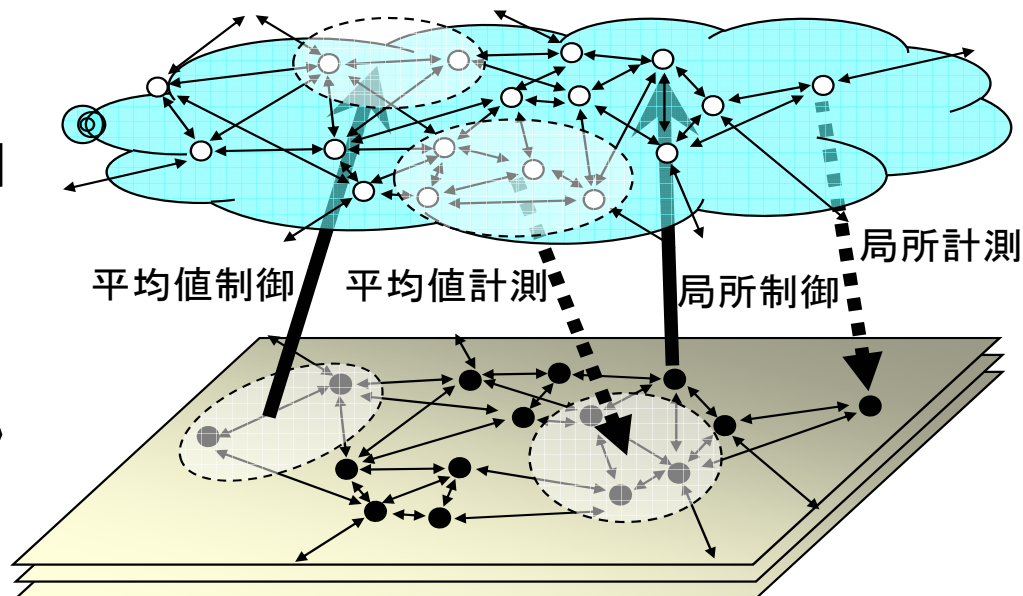
# 多分解能2重ネットワーク構造に基づくグローバル機能の実現

局所観測・制御による  
ヒートアイランド対策

免疫系階層モデルに基づく  
疾患状態の制御, . . .

物理ネットワーク（気象, 生体, . . .）

グローバル  
制御  
システム論  
物理NWから  
動的情報NW  
への架け橋  
（適合性）



グローバル  
制御  
情報論  
動的情報NW  
から物理NW  
への架け橋  
（整合性）

動的情報ネットワーク（計測・予測・制御ネットワーク）

## 多分解能表現に基づく動的情報ネットワーク設計

- ・ 多分解能計測 - どの分解能でいつどこ何を計測？
- ・ リアルタイム予測 - どの分解能でいつどこ何を予測？
- ・ 分散制御 - どの分解能でいつどこ何を制御？

Urban Heat Island      Immune Systems

Validation through  
Real Applications

Physical Network

Compatibility  
of Two NWs  
(Internal Model)



Consistency of  
Multi-resolution  
(State Resolution)

Information Network



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