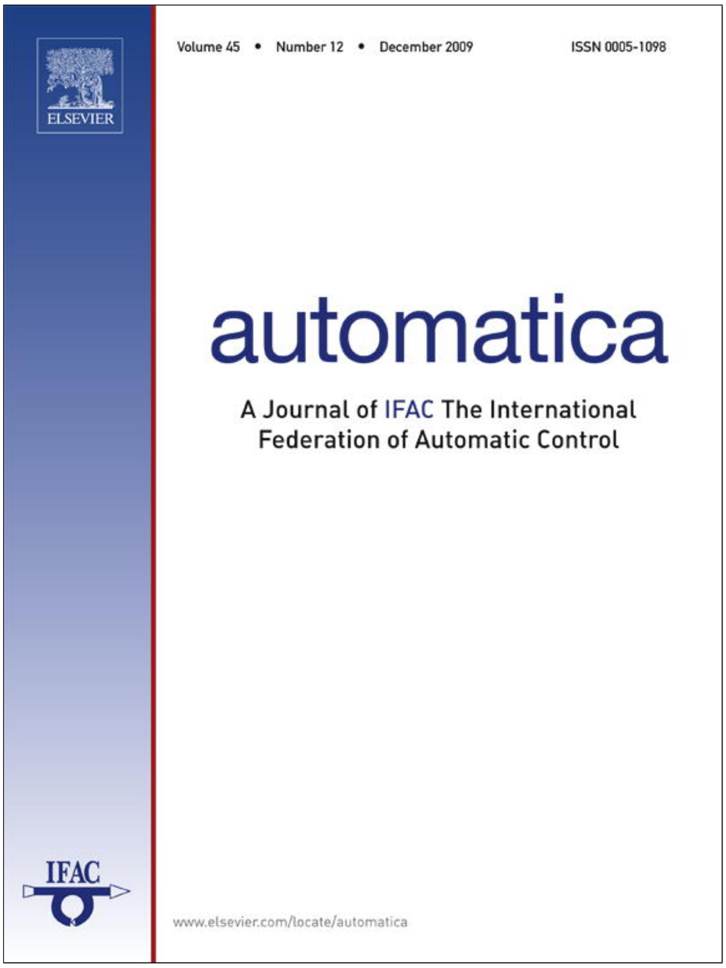
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Brief paper

Tradeoffs between quantization and packet loss in networked control of linear systems[☆]

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ABSTRACT

In this paper, we consider to derive the coarsest memoryless quantizer which can stabilize a single-input discrete-time linear time-invariant system with stochastic packet loss in the sense of stochastic quadratic stability. We show that the upper bound of the coarseness is strictly given by the packet loss probability and the unstable poles of the plants. We furthermore deal with permissible dead-zone width around the origin of the quantizers and time-varying finite quantizers in order to realize control using finite quantization steps.

1. Introduction

In recent years, networked control systems have been actively investigated in the field of control theory and one of the interests is to find the relationship between the permissible coarseness of transmitted signals for stabilization and the properties of plants. Some of the recent works on this topic include (Brockett & Liberzon, 2000; Elia & Mitter, 2001; Fu & Xie, 2005; Goodwin, Haimovich, Quevedo, & Welsh, 2004; Nair & Evans, 2004; Tatikonda & Mitter, 2004a; Tsumura & Maciejowski, 2003; Wong & Brockett, 1999). In particular, in Elia and Mitter (2001) a stabilization problem via quantized input signals is considered and the coarsest memoryless quantizer for stabilization of single-input discrete-time linear time-invariant systems is derived. A notable point is that the upper bound of the coarseness is given only by the unstable poles of the plants. This result is also extended to LQR type problems (Fu & Xie, 2005) and adaptive control problems (Hayakawa, Ishii, & Tsumura, 2009a,b).

Another problem we should deal with for networked control systems is the packet loss in data transmission. This problem arises when unreliable communication channels are used such as wireless networks or general-purpose channels. Clearly, losses of signals cause performance degradation or can make a closed-loop system unstable. Some research groups have dealt with this problem. LQ type control problems are considered in Imer, Yüksel, and Başar (2006), and H_∞ control approaches were proposed in Seiler and Sengupta (2005) and Ishii (2008a). Sinopoli et al. (2004), studied stabilization in state estimation problems under packet losses. In Elia (2005) and Ishii (2008b), the mean square stability of feedback control systems is investigated and the upper limit of loss probability is given in terms of the unstable poles of the plants. For the scalar case, this was shown in Hadjicostis and Touri (2002).

In spite of the above significant results showing the relationships between "the unstable poles of plants and the coarseness of quantization (Elia & Mitter, 2001)" and between "the unstable poles of plants and the packet loss probability (Elia, 2005; Ishii, 2008b)," in real communication channels, it is more realistic to assume that the channel contains both quantization and stochastic packet losses. A natural extension of our interests is on the relationship among the three properties above for such networked control systems.

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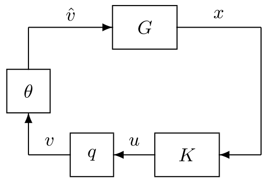

Fig. 1. Stabilization via quantized signals with stochastic packet losses.

This is our motivation of research and we investigate the coarsest memoryless quantizer for stabilization with stochastic packet losses. We show in particular that this upper bound of the coarseness is strictly given by the packet loss probability and the unstable poles of the plants (Section 2). As a consequence, we integrate and generalize the previous results of Elia and Mitter (2001) and those of Elia (2005) and Ishii (2008b).

In this paper, we furthermore deal with permissible dead-zone width around the origin of the quantizer for a stochastic version of practical stability (Section 3) and time-varying finite quantizers (Section 4) for realizing realistic quantizers which have finite quantization steps.

2. The coarsest quantizer for stabilization with stochastic packet losses

In this paper, we consider the following discrete-time linear system:

$$G:\ x(k+1) = Ax(k) + B\hat{v}(k), \tag{1}$$

where $x(k) \in \mathbb{R}^n$ is the state vector, $\hat{v}(k) \in \mathbb{R}$ is the control input, $A \in \mathbb{R}^{n\times n}$ and $B \in \mathbb{R}^{n\times 1}$. Assume that (A, B) is stabilizable and A is unstable.

We explain how the control signal is processed when it is transmitted from a controller to the control input of G according to Fig. 1. At first, the control signal $u(k)$ from the controller is quantized at the controller side before it is sent over a communication channel. The quantization is given by

$$v(k) = q(u(k)), \tag{2}$$

where $q(\cdot)$ is a memoryless quantizer and $u(k) \in \mathbb{R}$ is an ordinary analog control input generated by a static state feedback controller $K(\cdot)$.

In addition, we assume that packet losses occur with probability α at the input-side channel of the plant. In this paper, we employ a simple scheme where the packet loss sets $\hat{v}(k) = 0$,[1] and hence the system can be described as

$$x(k+1) = Ax(k) + B\theta(k)v(k), \tag{3}$$

where $\theta(k)$ is a 0–1 random variable with a probability distribution given by

$$\Pr(\theta(k) = i) = \begin{cases} \alpha, & i = 0, \\ 1-\alpha, & i = 1, \end{cases} \quad 0 \le \alpha < 1.$$

The reason why we deal with the case that the quantization is limited to the plant input side is that it is one of the basic setups. It is also a model where a large difference exists in the capacities for transmissions to and from the controller such as in a wireless-networked control system or a large-scale plant.

We next describe the stability we employ in this section. Consider the following discrete-time system:

$$x(k+1) = f(x(k), \theta(k)), \tag{4}$$

where $x(k) \in \mathbb{R}^n$ is the state, and $\theta(k) \in \{0, \dots, N-1\}$ represents the mode of the system. The mode is an independent and identically distributed stochastic process with probabilities $\alpha_i = \Pr(\theta(k) = i)$. The function $f(x, \theta)$ satisfies $f(0, \theta) = 0$ for arbitrary θ. Thus, the origin $x = 0$ of the system is an equilibrium point.

For this system we define the following stability:

Definition 2.1. For the system (4), the equilibrium point at the origin is stochastically quadratically stable if there exists a positive-definite function $V(x) = x^{\mathrm{T}}Px$ and a positive-definite matrix R such that

$$\begin{aligned} \Delta V &= E[V(x(k+1))|x(k)] - V(x(k)) \\ &\le -x(k)^{\mathrm{T}}Rx(k), \quad \forall x(k) \in \mathbb{R}^n. \end{aligned} \tag{5}$$

Remark 2.1. The condition (5) is sufficient for the origin of the system (4) to be mean square stable (see, e.g., Ji and Chizeck (1990)), i.e., for every initial state x_0,

$$\lim_{k\to\infty} E[\|x(k)\|^2|x_0] = 0. \tag{6}$$

The important point on the condition (5) is that the absolute averaged decreasing rate of a Lyapunov function V is larger than or equal to a quadratic form of x. Also we should note that another condition $\Delta V < 0$, $\forall x$, does not necessarily guarantee stability for the "stochastic" nonlinear system different from the case Elia and Mitter (2001). The matrix $R\,(> O)$ in (5) regulates the convergence rate of x and it is critical for the moment of x as shown in Proposition 3.1 and Theorem 3.1, Section 3, where we deal with a case that the quantizer has a dead-zone.

In this section, our objective is to find the coarsest quantizer $q(\cdot)$ which achieves stochastic quadratic stability for the system (3). The coarseness of a quantizer $q(\cdot)$ is defined as (Elia & Mitter, 2001)

$$d = \lim_{\epsilon\to 0}\sup \frac{\sharp u[\epsilon]}{-\ln \epsilon}, \tag{7}$$

where $\sharp u[\epsilon]$ denotes the number of levels that the quantizer $q(\cdot)$ has in the interval $[\epsilon, 1/\epsilon]$.

Elia and Mitter (2001) showed that the coarsest quantizer for the quadratic stabilization in the case of *no packet loss* is logarithmic and the coarsest expansion ratio $\rho_{\sup}$ (which is strictly defined later) is given by

$$\rho_{\sup} = \frac{\prod_i |\lambda_i^u| + 1}{\prod_i |\lambda_i^u| - 1}, \tag{8}$$

where λ_i^u represents the unstable poles of the plant. On the other hand, in Elia (2005) and Ishii (2008b), a necessary and sufficient condition on α for the mean square stabilizability in the case of *no quantization* is given as

$$\alpha < \alpha_{\sup} = \frac{1}{\prod_i |\lambda_i^u|^2}. \tag{9}$$

In this paper, we consider the effects of both *quantization and packet losses* and the natural extension of our interests is "What relationship between $\rho_{\sup}$, α and λ_i^u does there exist?" We provide a complete answer to this question in the following theorem, which unifies the results (8) and (9).

Theorem 2.1. *The coarsest quantizer $q_c(\cdot)$ with which the system (3) is stochastically quadratically stable is given as:*

$$q_c(u) = \begin{cases} v_i, & u \in \left(\dfrac{\rho_{\sup}+1}{2\rho_{\sup}}v_i,\ \dfrac{\rho_{\sup}+1}{2}v_i\right], \\ -v_i, & u \in \left[-\dfrac{\rho_{\sup}+1}{2}v_i,\ -\dfrac{\rho_{\sup}+1}{2\rho_{\sup}}v_i\right), \\ 0, & u = 0, \end{cases} \tag{10}$$

[1] More complex models for packet loss are possible; however, we deal with the simple and standard model in this paper.

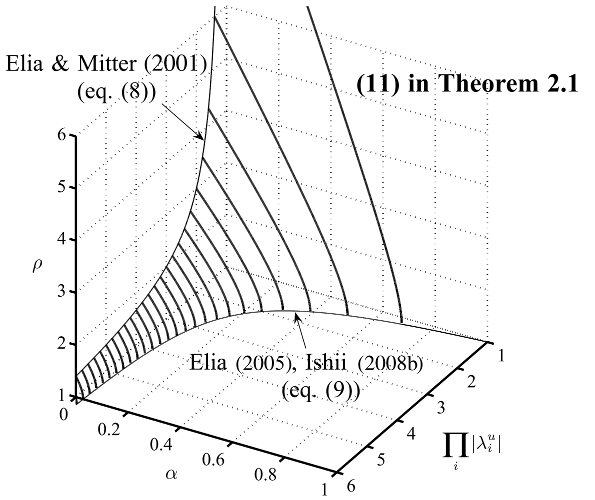

Fig. 2. The relationship between the product of unstable poles $\prod_i |\lambda_i^u|$, the loss probability α, and the coarseness ρ of the quantizer.

$$v_i = \rho_{\sup}^i v_0, \quad v_0 > 0,\ i \in \mathbb{Z},$$

$$\rho_{\sup} := \frac{\gamma_{\inf}+1}{\gamma_{\inf}-1}, \quad \gamma_{\inf} := \sqrt{\frac{1-\alpha}{\frac{1}{\prod_i |\lambda_i^u|^2} - \alpha}}, \tag{11}$$

where λ_i^u *is the unstable eigenvalue of* A *in* (3).

Remark 2.2. In general, logarithmic quantizers are given by $v_i = \rho^i v_0$ as in (10) where $\rho(> 1)$ represents the expansion ratio and a large ρ means a coarse quantizer. The "coarsest" quantizer (10) means that $\rho_{\sup}$ is the supremum of ρ for stochastic quadratic stability. That is, ρ, which achieves stochastic quadratic stability, should be smaller than $\rho_{\sup}$.

Theorem 2.1 generalizes two previous results (8) in Elia and Mitter (2001) and (9) in Elia (2005) and Ishii (2008b). Fig. 2 summarizes the results mentioned above. The relations in (8) and (9) are found on the $\prod_i |\lambda_i^u|$–ρ plane and the $\prod_i |\lambda_i^u|$–α plane, respectively. The curved surface represents (11) in Theorem 2.1, and it is easy to see that (11) unifies (8) and (9) and includes them as special cases. The result also shows a tradeoff between α and ρ, i.e., high packet loss probabilities require high resolution quantizers and vice versa for stability.

Proof. The outline of the proof is similar to that of Elia and Mitter (2001). We focus on the difference in the following, but also refer to some parts of the proof of Elia and Mitter (2001) in order to make the proof in a self-contained form.

At first, in order to find the coarsest quantization from u to v, we write as $v = u$ in form, and find P such that there exists an input u satisfying $\Delta V(x) < 0$ for a given x and a set $U(x, P, \alpha)$ of u for such P. By substituting (3) into the left-hand side of (5), we get

$$\begin{aligned}\Delta V &= E[V(Ax+\theta Bu)|x] - V(x)\\ &= x^{\mathrm{T}}(A^{\mathrm{T}}PA - P)x + 2(1-\alpha)x^{\mathrm{T}}A^{\mathrm{T}}PBu\\ &\quad + (1-\alpha)B^{\mathrm{T}}PBu^2 < 0.\end{aligned}$$

Then, $U(x, P, \alpha)$ can be given by

$$U(x, P, \alpha) = \{u \in \mathbb{R} | \Delta V < 0\} = \{u \in \mathbb{R} | u_- < u < u_+\}$$

$$u_\pm := -\frac{B^{\mathrm{T}}PA}{B^{\mathrm{T}}PB}x \pm \sqrt{\frac{x^{\mathrm{T}}Q(P,\alpha)x}{B^{\mathrm{T}}PB}}, \tag{12}$$

$$Q(P,\alpha) := \frac{P}{1-\alpha} - \frac{A^{\mathrm{T}}PA}{1-\alpha} + \frac{A^{\mathrm{T}}PBB^{\mathrm{T}}PA}{B^{\mathrm{T}}PB} > O, \tag{13}$$

and assume P satisfies the inequality (13).

According to Elia and Mitter (2001), x can be parametrized by

$$x = Q(P,\alpha)^{-1}A^{\mathrm{T}}PBp_1 + Q(P,\alpha)^{-1/2}p_2, \tag{14}$$

where $p_1 \in \mathbb{R}$ and $p_2 \in \mathbb{R}^n$ with $p_2 \perp Q^{-1/2}A^{\mathrm{T}}PB$, and therefore, u_- and u_+ can be written as

$$u_\pm = p_1\frac{B^{\mathrm{T}}PAQ(P,\alpha)^{-1}A^{\mathrm{T}}PB}{B^{\mathrm{T}}PB} \pm \sqrt{p_1^2\frac{B^{\mathrm{T}}PAQ(P,\alpha)^{-1}A^{\mathrm{T}}PB}{B^{\mathrm{T}}PB} + \|p_2\|^2\frac{1}{B^{\mathrm{T}}PB}}.$$

Then, for a given p_1, the allowable width $|u_+ - u_-|$ becomes minimum when $p_2 = 0$. Following Elia and Mitter (2001), we consider the case where x is in such a worst case 1-dimensional subspace $\{x | p_2(x) = 0\}$, and then we get

$$u_\pm = \tilde{p}_1(\hat{\gamma} \pm 1), \quad \tilde{p}_1 = \hat{\gamma}p_1,$$

$$\hat{\gamma} = \sqrt{\frac{B^{\mathrm{T}}PAQ(P,\alpha)^{-1}A^{\mathrm{T}}PB}{B^{\mathrm{T}}PB}}. \tag{15}$$

That is, the allowable inputs become a convex sector region where the origin is the vertex in the p_1–u plane.

Here, we should take care of the relationship between the stochastic quadratic stability (5) and $u_\pm$ as follows. The quantities $u_\pm$ are the boundaries of u satisfying $\Delta V < 0$. Therefore, for the actual quantizer in order to attain (5), we should introduce $\gamma > \hat{\gamma}$ and redefine both ends of the sector for u as $\tilde{u}_\pm = \tilde{p}_1(\gamma \pm 1)$ for the existence of $R > O$ in (5). However, for any u in $\tilde{u}_- \le u \le \tilde{u}_+$ with $\gamma > \hat{\gamma}$, there always exists R such that $\Delta V \le -x^{\mathrm{T}}Rx$, $\forall x$ from the definition of the coarseness (7). In the sense above, $u_\pm$ given in (15) are also the upper and lower bounds of possible u for a given x (i.e., $\tilde{p}_1$) in order to satisfy (5) and $\hat{\gamma}$ is the lower bound of the possible γ. Hereafter, we focus on these boundaries.

According to Elia and Mitter (2001), the coarsest quantizer of u is the following coarsest piecewise constant function in the sector in the p_1–u plane explained above:

$$q(u) = \begin{cases} v_i, & u \in \left(\frac{\rho+1}{2\rho}v_i, \frac{\rho+1}{2}v_i\right], \\ -v_i, & u \in \left[-\frac{\rho+1}{2}v_i, -\frac{\rho+1}{2\rho}v_i\right), \\ 0, & u = 0, \end{cases} \tag{16}$$

where $v_0 > 0$, $v_i = \rho^i v_0$, $i \in \mathbb{Z}$, and $\rho(P,\alpha) = \frac{\hat{\gamma}+1}{\hat{\gamma}-1}$. Moreover, the quantization density d is given as $d = \frac{2}{\ln \rho(P,\alpha)}$. Note that d is monotonically decreasing in ρ, and $\rho = \frac{\hat{\gamma}+1}{\hat{\gamma}-1}$ is also monotonically decreasing in $\hat{\gamma} > 1$, and then, the minimization of d reduces to that of $\hat{\gamma}$ as follows:

$$\gamma_{\inf} = \inf_{P>0,\ Q(P,\alpha)>O} \hat{\gamma}.$$

The above problem is equivalent to the minimization of γ subject to

$$A^{\mathrm{T}}PA - P - \left(1 - \frac{1+\alpha(\gamma^2-1)}{\gamma^2}\right) \times A^{\mathrm{T}}PB\left(B^{\mathrm{T}}PB\right)^{-1}B^{\mathrm{T}}PA < O, \quad P > O, \tag{17}$$

from (13) and (15).

From Lemma A.1 of Ishii (2008b), the condition for the existence of P and γ satisfying (17) is equivalent to the existence of a feedback gain F and γ such that

$$\|F(zI - A - BF)^{-1}B\|_\infty < \frac{\gamma}{\sqrt{1+\alpha(\gamma^2-1)}}$$

in a state feedback system. Note that the right-hand side above is monotonically increasing in $\gamma > 1$. Then, the lower bound of γ can be given by the minimization of the left-hand side. By employing Lemma 2.4 in Fu and Xie (2005), which gives the lower bound of H_∞ norm of the state feedback systems, we get $\gamma_{\inf} = \sqrt{\frac{1-\alpha}{\frac{1}{\prod_i |\lambda_i^u|^2}-\alpha}}$. ■

Remark 2.3. We remark that the proof above is first based on the condition $\Delta V < 0$. This is to obtain the critical bound $\rho_{\sup}$ on the expansion ratio ρ for the quantizer. However, as discussed in Remark 2.1, to guarantee stochastic quadratic stability when a quantizer with ρ smaller than $\rho_{\sup}$ is used, we must check the existence of the matrix $R > 0$ in (5). In the proof, we show that this holds true.

3. Dead-zone width design of quantizer for practical stabilization

The quantizer of the form (10) requires infinitesimal quantization levels around the origin. In order to keep the complexity of the signals sent over the channels reasonable, the quantizer should have a dead-zone around the origin though in a such case, asymptotic stability as (6) cannot be guaranteed. On the other hand, from the result in the previous section, the logarithmic quantizer is appropriate except for the neighborhood at origin. Thus, in this section, we consider practical stabilization of control systems using a logarithmic quantizer with dead-zone under packet losses.

The problem setting in this section follows that in the previous section and is as follows: The plant is (3) with stochastic packet losses where the loss probability α is assumed to satisfy (9). In this section, the quantizer $q_{\mathrm{dz}}(\cdot)$ is assumed to be memoryless and logarithmic with a dead-zone around origin and it can be expressed as

$$q_{\mathrm{dz}}(u) = \begin{cases} v_i, & u \in \left(\dfrac{\rho+1}{2\rho}v_i, \dfrac{\rho+1}{2}v_i\right], \\ -v_i, & u \in \left[-\dfrac{\rho+1}{2}v_i, -\dfrac{\rho+1}{2\rho}v_i\right), \\ 0, & u \in [-\nu, \nu], \end{cases} \tag{18}$$

$v_i = \rho^i v_0, \quad v_0 > 0,\ i \in \mathbb{Z}_{\geq 0},$

where the coarseness ρ of the quantizer satisfies $1 < \rho < \rho_{\sup}$. Note that the relationship between ν and v_0 is $\nu = \frac{\rho+1}{2\rho}v_0$. Moreover, $u(k)$ is given by $u(k) = Kx(k)$, $K = -\frac{B^{\mathrm{T}}PA}{B^{\mathrm{T}}PB}$, where $P \in \mathbb{R}^{n\times n}$ is the positive-definite matrix satisfying a Riccati type inequality:

$$A^{\mathrm{T}}PA - P - (1-\alpha)\left(1-\left(\frac{\rho-1}{\rho+1}\right)^2\right) \times A^{\mathrm{T}}PB(B^{\mathrm{T}}PB)^{-1}B^{\mathrm{T}}PA < O, \tag{19}$$

which is given by (17) and γ satisfying (17) with the relationship $\rho = \frac{\gamma+1}{\gamma-1}$.

Then, we further define a positive-definite matrix R and a positive number δ such as

$$R = P - A^{\mathrm{T}}PA + (1-\alpha)\left(1-\left(\frac{\rho-1}{\rho+1}\right)^2\right) \times A^{\mathrm{T}}PB(B^{\mathrm{T}}PB)^{-1}B^{\mathrm{T}}PA - \delta I, \tag{20}$$

where δ is chosen so that R is positive definite.

Remark 3.1. When $\nu \to 0$, the quantizer above goes to (16) in the previous section. Then the origin of the system (3) is stabilized in the sense of stochastic quadratic stability (5).

We next define the stability considered in this section.

Definition 3.1. For the system (4), the equilibrium point at the origin is mean square practically stable for a given $e > 0$ if for every initial state x_0,

$$\lim_{k\to\infty} E[\|x(k)\|^2|x_0] \leq e. \tag{21}$$

We should note that the quantizers are introduced to decrease the amount of information which is transferred over the channels. Therefore, the focus of the problem here is to find how large the dead-zone can be taken for a given stability specification e.

Remark 3.2. In Elia and Mitter (2001), a similar problem is considered for deterministic systems without packet losses. In such a case, R in (5) does not affect the relationship between e and the width ν of the dead-zone. On the other hand, in the stochastic systems considered in this paper, R plays an important role as we will explain later.

We now present the result as the following theorem:

Theorem 3.1. *Under the control law $v(k) = q_{\mathrm{dz}}(Kx(k))$ with the quantizer in (18), the origin of the system (3) is mean square practically stable for a given $e > 0$ if*

$$\nu \leq \sqrt{\frac{c}{\beta}},$$

where $c > 0$ is given as

$$c = \frac{\lambda_{\min}(P)\lambda_{\min}(R)}{\lambda_{\max}(P)}e,$$

and β is the minimum positive number such that

$$\beta\Phi - \bar{P} - \tau_1(\bar{P}_A - \bar{P} + \bar{R}) \geq 0, \quad \exists\tau_1 \geq 0, \tag{22}$$

$$\beta\Phi - \bar{P}_A - \tau_2(\bar{P}_A - \bar{P} + \bar{R}) \geq 0, \quad \exists\tau_2 \geq 0, \tag{23}$$

$$\Phi = diag[0\ \dots\ 0\ 1], \qquad \bar{P} = T^{\mathrm{T}}PT, \qquad \bar{R} = T^{\mathrm{T}}RT,$$
$$\bar{P}_A = \bar{A}^{\mathrm{T}}\bar{P}\bar{A}, \quad \bar{A} = T^{-1}AT.$$

Moreover, the matrix $T \in \mathbb{R}^{n\times n}$ is an invertible matrix defined by

$$T = \left[W \quad \frac{K^{\mathrm{T}}}{\|K^{\mathrm{T}}\|^2}\right], \tag{24}$$

and $W \in \mathbb{R}^{n\times(n-1)}$ is an arbitrary matrix satisfying $KW = O$.

Remark 3.3. The inequalities (22) and (23) are LMIs of τ_1, τ_2, and β and are thus convex. Therefore, the minimum β satisfying the conditions can be computed by efficient algorithms.

For the proof of this theorem, we first show the following lemma and proposition.

Lemma 3.1. *For the system (3), the quantizer (18) and a given positive number c, the following are satisfied:*

$$E[V(x(k+1))|x(k)] \leq c, \quad \forall x(k) \in L_V(c), \tag{25}$$

$$\Delta V = E[V(x(k+1))|x(k)] - V(x(k)) \leq -x(k)^{\mathrm{T}}Rx(k), \quad \forall x(k) \in \mathbb{R}^n \setminus L_V(c), \tag{26}$$

where $L_V(c)$ is the following level set: $L_V(c) = \{x \in \mathbb{R}^n | V(x) \leq c\}$, ν in the quantizer (18) is a positive number satisfying $\nu \leq \sqrt{\frac{c}{\beta}}$, and β is a solution of (22) and (23).

Proof. At first, consider the following transformation with T of (24): $x = T\left[r_1^{\mathrm{T}} \quad r_2\right]^{\mathrm{T}}$, where $r_1 \in \mathbb{R}^{n-1}$, $r_2 \in \mathbb{R}$. Note that $r_2 = u$ from the definition of T.

Next define $\mathcal{D} := \{x \in \mathbb{R}^n \mid -\nu \le r_2 \le \nu\}$. This represents the region in the state space such that the output of the quantizer becomes 0. The region $\mathcal{D}$ can be divided into the following two parts:

$$\mathcal{S} := \left\{x \in \mathcal{D} \mid x^{\mathrm{T}}(A^{\mathrm{T}}PA - P + R)x > 0\right\},$$
$$\overline{\mathcal{S}} := \left\{x \in \mathcal{D} \mid x^{\mathrm{T}}(A^{\mathrm{T}}PA - P + R)x \le 0\right\}.$$

Here the important region is $\mathcal{S}$ in order to show (25) and (26) are satisfied. The reason is the following. In the dead-zone free case, the quantization (18) for $\nu = 0$ satisfies $\Delta V \le -x^{\mathrm{T}}Rx$ for the whole space of x, and therefore, (26) is satisfied for $x(k) \in \mathbb{R}^n \setminus L_V(c)$. On the other hand, for the condition (25), it is enough to consider the region $\mathcal{S}$ of x in which the decrease of the Lyapunov function is not guaranteed.

If $x(k) \in \mathcal{D}$ at k, then, deterministically $x(k+1) = Ax(k)$ at $k+1$, and therefore, when $x(k) \in \mathcal{S}$, we have $x(k+1) = Ax(k) \in A\mathcal{S}$.

From the above consideration, the problem reduces to finding a condition on ν to guarantee $x^{\mathrm{T}}Px \le c, x^{\mathrm{T}}A^{\mathrm{T}}PAx \le c, \forall x \in \mathcal{S}$. Instead of x, we represent the above by r_1 and r_2 as

$$\begin{bmatrix} r_1^{\mathrm{T}} & r_2 \end{bmatrix} \bar{P} \begin{bmatrix} r_1 \\ r_2 \end{bmatrix} \le c, \qquad \begin{bmatrix} r_1^{\mathrm{T}} & r_2 \end{bmatrix} \bar{P}_A \begin{bmatrix} r_1 \\ r_2 \end{bmatrix} \le c,$$

$\forall r_1, r_2$ such that

$$\begin{bmatrix} r_1^{\mathrm{T}} & r_2 \end{bmatrix} (\bar{P}_A - \bar{P} + \bar{R}) \begin{bmatrix} r_1 \\ r_2 \end{bmatrix} > 0, \quad -\nu \le r_2 \le \nu. \tag{27}$$

From the convexity of the condition (27), it is enough to evaluate them at $r_2 = \nu$.

Now we consider the following variable transformation from c and r_1 to β and $\hat{r}_1$ such as $c = \nu^2\beta$ $(\beta > 0)$ and $r_1 = \nu\hat{r}_1$. We get the following by the definition of Φ:

$$\begin{bmatrix} \hat{r}_1^{\mathrm{T}} & 1 \end{bmatrix} \left(\beta\Phi - \bar{P}\right) \begin{bmatrix} \hat{r}_1 \\ 1 \end{bmatrix} \ge 0,$$

$$\begin{bmatrix} \hat{r}_1^{\mathrm{T}} & 1 \end{bmatrix} \left(\beta\Phi - \bar{P}_A\right) \begin{bmatrix} \hat{r}_1 \\ 1 \end{bmatrix} \ge 0,$$

$\forall \hat{r}_1$ such that

$$\begin{bmatrix} \hat{r}_1^{\mathrm{T}} & 1 \end{bmatrix} (\bar{P}_A - \bar{P} + \bar{R}) \begin{bmatrix} \hat{r}_1 \\ 1 \end{bmatrix} > 0.$$

By the S-procedure, this condition is equivalent to the existence of non-negative τ_1 and τ_2 satisfying the following:

$$\beta\Phi - \bar{P} - \tau_1(\bar{P}_A - \bar{P} + \bar{R}) \ge O,$$
$$\beta\Phi - \bar{P}_A - \tau_2(\bar{P}_A - \bar{P} + \bar{R}) \ge O.$$

Therefore, for ν satisfying $\nu \le \sqrt{\frac{c}{\beta}}$, (25) and (26) are satisfied. ∎

Proposition 3.1. *For the system (3) with given positive-definite matrices P and R, and a given number $c > 0$, suppose (25) and (26) are guaranteed. Then, for arbitrary x_0, the following holds:*

$$\lim_{k\to\infty} E[\|x(k)\|^2 | x_0] \le \frac{\lambda_{\max}(P)}{\lambda_{\min}(P)\lambda_{\min}(R)} c. \tag{28}$$

Proof. With (26), in the case of $x(k) \notin L_V(c)$, we have $\frac{E[V(x(k+1))|x(k)]}{V(x(k))} \le 1 - \frac{x(k)^{\mathrm{T}}Rx(k)}{x(k)^{\mathrm{T}}Px(k)} \le a$, where $a := 1 - \frac{\lambda_{\min}(R)}{\lambda_{\max}(P)}$ $(0 < a < 1)$. Then, we get

$$E[V(x(k+1))|x(k)] \le aV(x(k)), \quad \forall x(k) \in \mathbb{R}^n \setminus L_V(c).$$

On the other hand, in the case of $x(k) \in L_V(c)$, (25) holds. By unifying these inequalities, we obtain

$$E[V(x(k+1))|x(k)] \le aV(x(k)) + c, \quad \forall x(k) \in \mathbb{R}^n.$$

For $k = 0$ and $k = 1$, the above becomes $E[V(x(1))|x_0] \le aV(x_0) + c$ and $E[V(x(2))|x(1)] \le aV(x(1)) + c$, respectively. By taking expectations of both sides, we get

$$\begin{aligned} E[V(x(2))|x_0] &\le aE[V(x(1))|x_0] + c \\ &\le a^2V(x_0) + ac + c. \end{aligned}$$

Thus, it follows that $E[V(x(k))|x_0] \le a^kV(x_0) + (a^{k-1} + \cdots + a + 1)c$ and then,

$$E[V(x(k))|x_0] \le a^kV(x_0) + \frac{c}{1-a}\left(1 - a^k\right).$$

Using a property of symmetric matrices, we have

$$\lambda_{\min}(P)\|x\|^2 \le x^{\mathrm{T}}Px \le \lambda_{\max}(P)\|x\|^2$$

for any x, and hence we obtain

$$\begin{aligned} E[\|x(k)\|^2|x_0] \le\ & \frac{\lambda_{\max}(P)}{\lambda_{\min}(P)} a^k \|x_0\|^2 \\ & + \frac{c}{\lambda_{\min}(P)(1-a)}\left(1 - a^k\right). \end{aligned}$$

By taking the limit as $k \to \infty$, (28) is derived. ∎

Remark 3.4. Different from the conventional method by LaSalle's invariance principle, Proposition 3.1 assures convergence on a set regardless of the sign of ΔV inside the level set $L_V(c)$. For continuous-time stochastic systems, this type of stability was discussed by Deng, Krstic, and Williams (2001). The result above is its discrete-time version.

Remark 3.5. From Proposition 3.1, the upper bound of the convergence radius (28) is given by c, P, and R. It is obvious that the index c directly controls the convergence property, and moreover, R also plays an important part in order to make the radius of convergence small.

The proof of Theorem 3.1 follows in a straightforward way by combining Lemma 3.1 and Proposition 3.1.

4. The use of time-varying finite quantizers

So far, we have considered static quantizers having an infinite number of output levels. In this section, we study the case with time-varying ones with finite levels.

First, we design the quantizer as in Section 3. We introduce a truncated version of the quantizer given in (18) as follows: Given $\upsilon_0 > 0$ and $N \in Z_+$, let

$$q_{\upsilon_0}(u) := \begin{cases} \upsilon_i, & u \in \left(\dfrac{\rho+1}{2\rho}\upsilon_i, \dfrac{\rho+1}{2}\upsilon_i\right], \\ -\upsilon_i, & u \in \left[-\dfrac{\rho+1}{2}\upsilon_i, -\dfrac{\rho+1}{2\rho}\upsilon_i\right), \\ 0, & u \in [-\nu, \nu], \end{cases}$$
$$\upsilon_i = \rho^i\upsilon_0, \ i = 0, 1, \ldots, N-1, \tag{29}$$

where $\nu = (\rho+1)\upsilon_0/2\rho$. The control input $v(k)$ is then given by

$$v(k) = q_{\upsilon_0(k)}(Kx(k)), \tag{30}$$

where the time-varying parameter $\upsilon_0(k)$ is to be specified. The parameter $\upsilon_0(k)$ changes the domain and the step widths of the quantization dynamically. That is, dynamic quantization is possible according to the magnitude of the input signal. Note that this quantizer takes only a finite number $2N + 1$ of quantization levels at each time. Moreover, the bit-rate necessary for updating $\upsilon_0(k)$ is finite. Therefore, the proposed quantization method can be realized using a finite capacity. This type of a quantization method was first introduced in Brockett and Liberzon (2000).

One assumption we make in this section is that the coder in Fig. 1 knows the value of $\theta(k)$ at time $k+1$. This may be realized by an acknowledgement message sent from the decoder if, e.g., more power is available there for transmission (e.g., Imer et al. (2006) and Ishii (2008a)). Another approach is to add an observer-like system on the sensor side to estimate the input applied (Sahai & Mitter, 2006). If this assumption fails to hold, a more conservative design would be necessary.

The stability definition used in this section is as follows.

Definition 4.1. The closed-loop system under the control law in (30) is said to be *stable with probability* 1 (*w.p.* 1) if $\|x(k)\| \to 0$ as $k \to \infty$ w.p. 1.

We provide a preliminary result. At first, take N large enough that

$$N \geq \log_\rho \frac{F_0}{\sqrt{1-\frac{\lambda_{\min}(\tilde{R})}{\lambda_{\max}(P)}}},$$

where

$$F_0 := \frac{\sqrt{\beta}(\rho+1)\,|\eta_+|}{2\sqrt{\lambda_{\min}(P)}\|\tilde{Q}^{-1}A^{\mathrm{T}}PB\|},$$

$$\tilde{R} := \frac{1}{1-\alpha}\left[R+\alpha(A^{\mathrm{T}}PA-P)\right],$$

and β is the value given in Theorem 3.1. Let

$$c_1(v_0) := \left(\frac{\rho+1}{2\rho}v_0\right)^2\beta,$$

$$c_2(v_0) := \lambda_{\min}(P)\left[\frac{\|\tilde{Q}^{-1}A^{\mathrm{T}}PB\|v_{N-1}}{|\eta_+|}\right]^2,$$

where

$$\tilde{Q} := \left(\frac{\rho-1}{\rho+1}\right)^2\frac{A^{\mathrm{T}}PBB^{\mathrm{T}}PA}{B^{\mathrm{T}}PB}+\frac{\delta}{1-\alpha}I,$$

$$\eta_\pm := -\frac{B^{\mathrm{T}}PA\tilde{Q}^{-1}A^{\mathrm{T}}PB}{B^{\mathrm{T}}PB} \pm \sqrt{\frac{B^{\mathrm{T}}PA\tilde{Q}^{-1}A^{\mathrm{T}}PB}{B^{\mathrm{T}}PB}}.$$

A result similar to Lemma 3.1 holds as shown in the next lemma.

Lemma 4.1. *Given $v_0 > 0$, under the control law $v(k) = q_{v_0}(Kx(k))$, the following hold:*

$$E[V(x(k+1))|x(k)] \leq c_1(v_0), \quad \forall x(k) \in L_V(c_1(v_0)), \tag{31}$$

$$\Delta V = E[V(x(k+1))|x(k)] - V(x(k)) \leq -x(k)^{\mathrm{T}}Rx(k), \quad \forall x(k) \in L_V(c_2(v_0)) \setminus L_V(c_1(v_0)). \tag{32}$$

Proof. In the notation of c_1 and c_2, we omit v_0 for simplicity. By Lemma 3.1, clearly, (32) holds for any $x(k) \notin L_V(c_1)$ and also (31) holds. By the choice of N, we have $c_1 < c_2$, that is, the set $L_V(c_2) \setminus L_V(c_1)$ is nonempty. Hence, we must show that with the truncated quantizer (29), the inequality (32) holds for each $x(k) \in L_V(c_2) \setminus L_V(c_1)$. By following an argument similar to that in the proof of Lemma 3.1, we can show that for a fixed $v_i \neq 0$, if $x(k)$ is of the form

$$x(k) = \tilde{Q}^{-1}A^{\mathrm{T}}PBp_1 + \tilde{Q}^{-1/2}p_2, \tag{33}$$

where $p_1 \in \left[\eta_-^{-1}v_i, \eta_+^{-1}v_i\right]$ and $p_2 \in \mathbb{R}^n$ with $p_2 \perp \tilde{Q}^{-1/2}A^{\mathrm{T}}PB$, then (32) holds. Note here that $\eta_\pm < 0$ because A is an unstable matrix.

It now follows that the set of all states x that are outside of $L_V(c_1)$ and can be written in the form in (33) for some $i \in \{0, \ldots, N-1\}$ contains the ball with center 0 and radius $\|\tilde{Q}^{-1}A^{\mathrm{T}}PB\|v_{N-1}/|\eta_+|$. The largest level set contained in this ball is $L_V(c_2)$. Thus, we have shown (32). ■

Now, the control protocol for the time-varying quantization scheme of (30) is as follows: Given a positive scalar $R_0 > 0$, suppose the initial state satisfies $\|x(0)\| \leq R_0$. There are two auxiliary time-varying parameters $v_0(k)$ and $w(k)$ used in the scheme. Let the initial value of v_0 be

$$v_0(0) = \sqrt{\frac{\lambda_{\max}(P)}{\lambda_{\min}(P)}}\frac{|\eta_+|R_0}{\|\tilde{Q}^{-1}A^{\mathrm{T}}PB\|\rho^{N-1}}.$$

Then, let

$$v_0(k+1) = \begin{cases} F_0\rho^{-N}v_0(k), & \text{if } (\theta(k), w(k)) = (1,1), \\ \sqrt{1-\frac{\lambda_{\min}(\tilde{R})}{\lambda_{\max}(P)}}\,v_0(k), & \text{if } (\theta(k), w(k)) = (1,0), \\ \sqrt{\frac{\lambda_{\max}(A^{\mathrm{T}}PA)}{\lambda_{\min}(P)}}\,v_0(k), & \text{if } \theta(k) = 0, \end{cases}$$

and let

$$w(k) = \begin{cases} 1, & \text{if } x(k) \in L_V(c_1(v_0(k))), \\ 0, & \text{if } x(k) \notin L_V(c_1(v_0(k))). \end{cases}$$

What need to be transmitted over the channel at each time k are the index of the quantized signal $v(k)$ and the binary signal $w(k)$. In both the coder and the decoder, $v_0(k)$ can be constructed because $\theta(k)$ is available on both sides.

The next theorem is the main result of the section and characterizes the stabilization of the system in (3).

Theorem 4.1. *If $\rho \in (1, \rho_{\sup})$ and $\alpha \in (0, \alpha_{\sup})$ are sufficiently small that*

$$\left[1-\frac{\lambda_{\min}(\tilde{R})}{\lambda_{\max}(P)}\right]^{1-\alpha}\left[\frac{\lambda_{\max}(A^{\mathrm{T}}PA)}{\lambda_{\min}(P)}\right]^{\alpha} < 1, \tag{34}$$

then the closed-loop system under the control law specified above is stable with probability 1.

Proof. We first claim that $x(k) \in L_V(c_2(v_0(k)))$, $\forall k$. We show this by induction. By the assumption on $x(0)$ and the choice of $v_0(0)$, we have $x(0) \in L_V(c_2(v_0(0)))$.

Now, assume $x(k) \in L_V(c_2(v_0(k)))$. Suppose $\theta(k) = 0$. Then, the control input is $\hat{v}(k) = 0$. Hence,

$$V(x(k+1)) = x^{\mathrm{T}}(k)A^{\mathrm{T}}PAx(k) \leq c_2(v_0(k+1)).$$

That is, $x(k+1) \in L_V(c_2(v_0(k+1)))$.

Next, if $\theta(k) = 1$, there are two cases. The first case is when $x(k) \notin L_V(c_1(v_0(k)))$. We can show by (32) in Lemma 4.1 that

$$V(x(k+1)) \leq -\frac{1}{1-\alpha}x^{\mathrm{T}}(k)\left[\alpha A^{\mathrm{T}}PA - P + R\right]x(k) \leq c_2(v_0(k+1)).$$

The second case is when $x(k) \in L_V(c_1(v_0(k)))$. Then, by definition, $v_0(k+1) = F_0\rho^{-N}v_0(k)$, which implies $c_2(v_0(k+1)) = c_1(v_0(k))$. By construction of the quantizer, it is clear that if $v(k) \neq 0$, then $V(x(k+1)) \leq V(x(k)) \leq c_1(v_0(k))$. If $v(k) = 0$, then applying Lemma 4.1 to this case yields $E[V(x(k+1))|x(k)] = V(x(k+1)) \leq c_1(v_0(k))$. Hence, the claim is proved.

Now, to show stability, it is sufficient to prove $v_0(k) \to 0$ as $k \to \infty$ w.p. 1. In doing so, let $\bar{v}_0(k+1) = \psi_{\theta(k)}\bar{v}_0(k)$ with $\bar{v}_0(0) = v_0(0)$, where $\psi_0 := [\lambda_{\max}(A^{\mathrm{T}}PA)/\lambda_{\min}(P)]^{1/2}$ and $\psi_1 := [1-\lambda_{\min}(\tilde{R})/\lambda_{\max}(P)]^{1/2}$. Due to the size of N, it follows that $F_0\rho^{-N} \leq \psi_1$. Thus, $v_0(k) \leq \bar{v}_0(k)$ for all k.

Hence, we have $\bar{v}_0(k) = \prod_{l=0}^{k-1}\psi_{\theta(l)}\bar{v}_0(0)$. In what follows, we show that $\prod_{l=0}^{k-1}\psi_{\theta(l)} \to 0$ as $k \to \infty$, w.p. 1. By the assumption

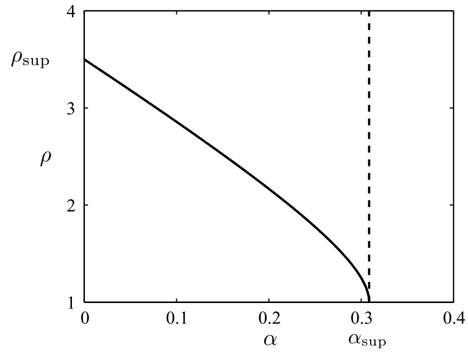

Fig. 3. The coarseness ρ of the logarithmic quantizer vs the loss probability α. The dashed line is the upper limit $\alpha_{\rm sup} = 1/\prod_i |\lambda_i^u|^2 = 0.31$ of α.

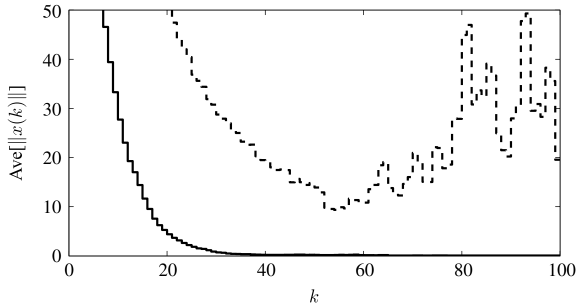

Fig. 4. Ave[$\|x(k)\|$] vs time (solid line: $\rho = 1.69$, dashed line: $\rho = 3.33$).

in (34), we have $E[\log_2 \psi_{\theta(l)}] = \alpha \log_2 \psi_0 + (1-\alpha)\log_2 \psi_1 < 0$, $\forall l$. The law of strong numbers implies

$$\frac{1}{k}\sum_{l=0}^{k-1} \log_2 \psi_{\theta(l)} \to E[\log_2 \psi_{\theta(l)}] < 0, \quad k \to \infty, \text{ w.p. } 1.$$

This in turn yields (Tatikonda & Mitter, 2004b)

$$\prod_{l=0}^{k-1} \psi_{\theta(l)} = 2^{k[1/k \sum_{l=0}^{k-1} \log_2 \psi_{\theta(l)}]} \to 0, \quad k \to \infty, \text{ w.p. } 1.$$

Thus, stability is now shown. ■

The quantized control scheme in this section has a simple structure. Especially, the decoder needs to calculate only $v_0(k)$, which is a scalar. This is in contrast to the time-varying, finite data rate control approach proposed in Tatikonda and Mitter (2004b). We however note that the parameters ρ and α satisfying the condition (34) can be conservative.

5. Numerical example

In this section, we present a numerical example to demonstrate the utility of the proposed quantizer design.

As the system (3), we considered the second-order system given by

$$x(k+1) = \begin{bmatrix} 0 & 1 \\ 1.8 & -0.3 \end{bmatrix} x(k) + \theta(k) \begin{bmatrix} 0 \\ 1 \end{bmatrix} v(k).$$

The system is unstable and has two poles 1.2 and −1.5.

In Fig. 2, this system corresponds to the cross section obtained by cutting the surface at $\prod_i |\lambda_i^u| = 1.8$. This cross section is depicted in Fig. 3. It is obvious that we need to select a communication channel with a packet loss rate $\alpha < \alpha_{\rm sup} \simeq 0.31$. Here, we chose a channel with $\alpha = 0.25$. This in turn requires that the quantizer has its parameter as $\rho < \rho_{\rm sup} \simeq 1.78$; we took $\rho = \rho_{\rm sup} \times 0.95 \simeq 1.69$.

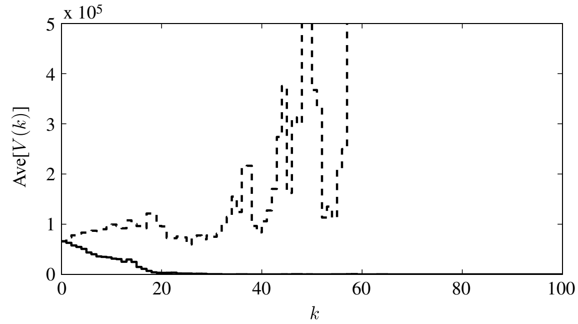

Fig. 5. Ave[$V(k)$] vs time (solid line: $\rho = 1.69$, dashed line: $\rho = 3.33$).

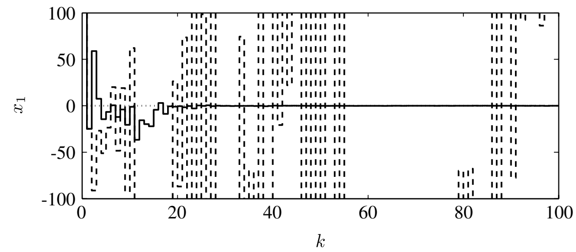

Fig. 6. Sample paths I: State x_1 vs time (solid line: $\rho = 1.69$, dashed line: $\rho = 3.33$).

In the setting above, there exists a $P > 0$ which satisfies (19). We can then obtain $R > 0$ such that (20) holds with some δ. So, we found the matrices P and R as

$$P = \begin{bmatrix} 3.264 & -1.180 \\ -1.180 & 3.313 \end{bmatrix}, \qquad R = \begin{bmatrix} 0.043 & -0.010 \\ -0.010 & 0.033 \end{bmatrix}$$

with $\delta = 0.01$, and the state feedback $K = [-1.800\ 0.656]$. It then follows that with the quantizer $q(\cdot)$ without a dead-zone, the closed-loop system is mean square stable.

Next, we focused on stabilizing the system in the sense of (21). Specifically, we fixed $e = 200$. From Theorem 3.1, we need to use a dead-zone width of $\nu \le \sqrt{\frac{c}{\beta}} =: \nu_{\max} \simeq 0.065$. With $\nu = 0.062 = \nu_{\max} \times 0.95$ and the random initial condition $x(0)$ satisfying $\|x_0\|_2 = \sqrt{2} \times 100$, we computed the time responses under packet losses 10^4 times. For comparison, we also performed a quantizer design without consideration of packet losses based on Elia and Mitter (2001), which gives a condition $\rho < 3.50 =: \rho_{\rm esup}$. Then, we set a quantizer having $\rho = \rho_{\rm esup} \times 0.95 \simeq 3.33$ and similarly simulated the time responses 10^4 times with the same initial conditions and packet loss sequences.

Figs. 4 and 5 show the averages of $\|x(k)\|$ and $V(k)$ of 10^4 samples, respectively. From them, we can see that the closed-loop system with $\rho = 1.69$ taking account of the packet losses goes very close to zero on average while the system designed by the conventional method with $\rho = 3.33$ does not and in fact the average of $V(k)$ diverges in this case.

Figs. 6 and 7 show two typical sample paths of the trajectories of $x_1(k)$ with $\rho = 1.69$ and $\rho = 3.33$. In Fig. 6, we observe that the closed-loop system with $\rho = 1.69$ converges close to zero and the system with $\rho = 3.33$ diverges, though the probability of such a case is not necessarily high. On the other hand, in Fig. 7, we present sample paths of usual cases where both trajectories converge. In this case, however, we can still observe that the convergence rate with $\rho = 1.69$ is much better than that with $\rho = 3.33$ and this fact shows the importance of ρ and the influence of packet losses.

6. Conclusion

In this paper, we have considered the stabilization problem of a linear system via quantized feedback with stochastic packet losses.

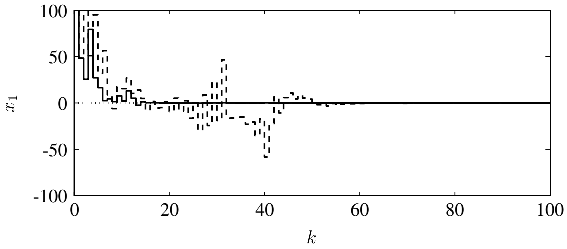

Fig. 7. Sample paths II: State x_1 vs time (solid line: $\rho = 1.69$, dashed line: $\rho = 3.33$).

We have presented the coarsest quantizer which can achieve stochastic quadratic stability. In particular, we have shown that the coarseness of the coarsest quantizer is strictly given by the packet loss probability and the unstable poles of the plant. Moreover, we have clarified the tradeoff between the level of quantization and the packet loss probability. In order to make the scheme more practical, we have also studied the cases with a dead-zone and with time-varying finite quantizers.

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