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Design and Stability of Optimal Frequency Control in Power Networks: A Passivity-based Approach

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Abstract— **Renewable energy generators give rise to large and frequent supply-demand power imbalances in modern power systems. In this volatile environment, secondary frequency control (SFC) is becoming a critical functionality of real-time operations and should be now carried out with higher economic efficiency. Motivated by that, in this paper we design an optimal frequency control (OFC) architecture that can be adopted in lieu of the current Automatic Generation Control (AGC) scheme enabling generators and demand response (DR) units to jointly carry out optimal frequency regulation with minimum generation cost and user disutility. The OFC algorithm can improve the economic efficiency of the secondary control layer by allowing the secondary control set-points to converge online to their optimal values. Interestingly, we show that the overall system composed of the physical network and OFC algorithm dynamics is passive. By leveraging this passivity property we establish global asymptotic stability of the equilibrium of the overall system. Our passivity-based methodology is scalable and computationally efficient and can be used to establish guarantees for the performance of a power network that adopts the proposed OFC algorithm particularly attractive for large-scale applications.**

I. Introduction

Environmental concerns pertaining to carbon emissions are the predominant drivers behind the increasing penetration of renewable energy resources (RERs) in power systems. On the downside, the highly variable and intermittent nature of these resources induces frequent and severe power fluctuations in the generation while at the same time, the overall reduced system inertia due to the retirement of conventional units has the potential to enhance the impact of these fluctuations on frequency quality. These high-impact frequency disturbances can greatly challenge both the security and reliability of power systems. To this end, SFC, which is the primary means for counterbalancing frequency disturbances, is expected to be heavily deployed in the foreseeable future.

In modern power systems with high penetration of renewable energy resources (RERs), automatic generation control (AGC) would have to offset large power mismatches between supply and demand to effectively carry out frequency regulation. For this reason, various experts argue that the economic efficiency of this fast power balancing process warrants now additional consideration and should not be overlooked any more. The economic efficiency of the AGC is dependent primarily on the way the area control error (ACE) is disaggregated to generators in real-time. AGC disaggregates the ACE to generators by employing a simple rule that relies on participation factors [1]. Despite the fact that these participation factors are designed from an economics perspective, they are inadequate in effectively ameliorating the economic efficiency at the secondary control (SC) layer. This can be inferred from the fact that the participation factors are updated every 5-15 mins, coincident with Economic Dispatch (ED), whereas power grid conditions deviate significantly from the nominal ones during the intra-dispatch time intervals. Hence, as participation factors are updated on a much slower timescale than the one dictating the variations in system conditions, the AGC becomes highly inefficient from an economics point of view [2]. This is even more urgent, as the growing presence of DR units will introduce additional time variations. Therefore a systematic methodology for optimal secondary frequency control is becoming highly necessary.

Related Work. In order to improve the economic efficiency of the SC layer, recent work has aimed at breaking the separation between fast frequency control and slow optimization-based ED and combining the two together. In [3], the authors proposed a decentralized control algorithm that can result in frequency regulation with minimal generation cost and benchmarked it against a centralized OFC algorithm. In [2], the authors probed the connection between AGC and ED from an optimization point of view and devised two modified AGC schemes with higher economic efficiency. In [4], the authors introduced one fully decentralized and one distributed frequency control algorithm both of which, can jointly carry out frequency regulation and ED. In [5], the authors proposed a modification in the current generation control system in order to improve its overall economic efficiency by incorporating terms that spring from the KKT conditions of the OFC problem. In [6], the authors examined the stability and optimality properties of distributed secondary frequency control schemes under general classes

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of generation and demand control dynamics. Finally, in [7], the authors assessed the impact of communication network failures on the performance of a consensus-based distributed optimal frequency control scheme similar to the one in [4].

Contributions. A universal element of the optimization methods in [3], [2], [4], [5], [6], [7] is the slow convergence rate. In practice, this particular aspect can pose a hurdle to realizing high economic efficiency through the proposed OFC schemes. This can be justified by the fact that the generation and demand power commands, under the proposed OFC algorithms, converge to their optimal values at a relatively slow rate compared to the fast frequency regulation dynamics. To get around this shortcoming among others, we make the following contributions: 1) We employ a variant of Newton's method (used in our previous work [8] and [9]) to design a real-time optimization algorithm for attaining solution to the OFC problem online. Such method is best-suited for the OFC problem due to its fast convergence rate. Specifically, our proposed algorithm enables the generation and DR power set-points to converge to their optimal values significantly faster than with previously proposed first-order methods, 2) We introduce an elegant methodology based on passivity theory for proving in a compositional manner that the overall system composed of the physical network and the OFC algorithm is passive and has an asymptotically stable equilibrium. Particularly attractive merits of our methodology are its computational efficiency and thus scalability and its flexibility for handling more general classes of system dynamics.

The rest of the paper is organized in the following way. Section II reviews basic power systems models and discusses the current AGC. Sections III and IV embody the main results of the paper. In particular, Section III states the OFC problem formulation and presents the real-time OFC algorithm. Section IV presents the compositional stability analysis based on passivity theory. Finally, Section V concludes the paper with some remarks.

II. Modeling and Background

A. Power System Model

Consider a power system comprised of n buses with its physical topology modeled by a connected undirected graph $\mathcal{G} = (\mathcal{N}, \mathcal{E})$ where, $\mathcal{N}$ denotes the set of buses and $\mathcal{E} \subseteq \mathcal{N} \times \mathcal{N}$ the set of transmission lines connecting the buses. Concretely, a line connecting a bus i with a bus j is denoted by $(i, j) \in \mathcal{E}$. In our analysis, $\mathcal{G}$ is the set of generators and $\mathcal{D}$ the set of demand response (DR) units. The set of generator buses is described by $\mathcal{N}_G$ and the set of load buses by $\mathcal{N}_L$ such that $\mathcal{N} \triangleq \mathcal{N}_G \cup \mathcal{N}_L$. We adopt a linearized DC power flow model and make the following typical assumptions:

- the voltage magnitudes are fixed
- the resistances of the transmission lines are negligible, i.e $R_{ij} = 0,\ \forall (i, j) \in \mathcal{E}$.
- the voltage angle differences are small
- the reactive power flows are neglected.

These assumptions are well-justified for power transmission networks as corroborated by [1]. In view of them, the linearized dynamics of the bus frequencies ω_i and voltage angles δ_i around their equilibria can be succinctly written as:

$$M_i\dot{\omega}_i = -D_i\omega_i + \sum_{j\in\mathcal{G}_i}(P_{M,j} + P_{St,j}) - \sum_{j\in\mathcal{D}_i} P_{D,j} - P_{L,i} - \sum_{j\in\mathcal{N}} T_{ij}(\delta_i - \delta_j),\ \ i \in \mathcal{N}_G \quad (1)$$

$$D_i\dot{\delta}_i = -\sum_{j\in\mathcal{D}_i} P_{D,j} - P_{L,i} - \sum_{j\in\mathcal{N}} T_{ij}(\delta_i - \delta_j),\ \ i \in \mathcal{N}_L \quad (2)$$

$$\dot{\delta}_i = \omega_i,\ \ i \in \mathcal{N} \quad (3)$$

Equations (1)-(3) describe the swing dynamics where M_i are the inertias of the generators, D_i the damping coefficients, $P_{M,j}$ the power outputs of generators, $P_{St,j}$ is the power output of local storage devices, $P_{D,j}$ the power consumption references for the DR units and $P_{L,i}$ the inflexible load. In addition, the constants T_{ij} are the transmission coefficients which can be defined as $T_{ij} = (V_i^* V_j^*)/X_{ij}$ where X_{ij} is the reactance of the transmission line (i, j) and V_i^*, V_j^* are the voltage magnitudes at equilibrium. The variables $P_{M,j}, P_{D,j}, P_{L,i}$ also denote deviations from the equilibrium.

Each generator is assumed to be equipped with a governor controller. By adopting a first-order model for the combined governor and prime-mover dynamics, the mechanical power output of each generator can be expressed as:

$$\dot{P}_{M,i} = \tau_i^{-1}(P_{G,i} - P_{M,i} - R_i^{-1}\omega_j),\ \ i \in \mathcal{G} \quad (4)$$

where ω_j corresponds to the frequency of the bus that the generator is connected to, τ_i is the time constant and $P_{G,i}$ the power set-point for the generator.

B. Conventional Automatic Generation Control

Frequency regulation has been traditionally carried out by the Automatic Generation Control (AGC) scheme whose objective is to drive the frequency and tie line flows back to their nominal values shortly after a power imbalance [1]. For this scope, the AGC uses the area control error (ACE) which, for a particular area a, can be defined as:

$$ACE^{(a)} = B^{(a)}\overline{\omega}^{(a)} + \sum_{(i,k)\in\mathcal{T}_f^{(a)}} P_{ik} - \sum_{(k,i)\in\mathcal{T}_t^{(a)}} P_{ki} \quad (5)$$

where $\overline{\omega}^{(a)}$ corresponds to the average frequency of the generator buses and P_{ik} to the power flowing through the tie line $(i, k) \in \mathcal{T}_f^{(a)}$ *from* area a to other areas and P_{ki} to the power flowing through the line $(k, i) \in \mathcal{T}_t^{(a)}$ *to* area a from other adjacent areas. The constant $B^{(a)}$ refers to the area's frequency bias factor and is given by:

$$B^{(a)} = \sum_{i\in\mathcal{N}^{(a)}} D_i + \sum_{i\in\mathcal{G}^{(a)}} R_i^{-1} \quad (6)$$

Practically speaking, the ACE reflects the aggregate power mismatch between supply and demand in a given area. The current AGC logic mandates that the integral of ACE is transmitted to the all units that provide regulation scaled each time by a coefficient that is equal to the participation factor of the particular unit. On the receiving end, each regulating unit uses this information to compute its new power reference:

$$\dot{P}_{G,i} = -pf_i \cdot k \cdot ACE^{(a)}, \quad i \in \mathcal{G}^{(a)} \tag{7}$$

where k is a control gain. According to [1], the ISOs compute the participation factors pf_i through the formula:

$$pf_i \triangleq \Big[\sum_{j\in\mathcal{G}^{(a)}} (1/C''_{G,j}(P^*_{G,j}))\Big]^{-1}\Big(1/C''_{G,i}(P^*_{G,i})\Big) \tag{8}$$

where $C_{G,i} \in C^2$, i.e the cost functions are twice differentiable. The term $P^*_{G,i}$ corresponds to the more recent dispatch. As the power system operating conditions during the intra-dispatch intervals vary largely and rapidly the above static AGC logic based on participation factors that are updated every $5-15$ mins leads to low economic efficiency.

III. Problem Formulation and Optimal Frequency Control Algorithm

A. Optimal Frequency Control Problem

Our aim here is to improve the economic efficiency associated with the secondary control layer. To this end, we seek to bring cost-optimality in the fast power balancing process that is performed by this control layer. To carry out this, we formulate the OFC problem as:

OFC

$$\underset{P_G,P_D}{minimize} \quad \sum_{i\in\mathcal{G}} C_{G,i}(P_{G,i}) - \sum_{i\in\mathcal{D}} U_{D,i}(P_{D,i}) \tag{9a}$$

$$\text{subject to} \quad \sum_{i\in\mathcal{G}} P_{G,i} - \sum_{i\in\mathcal{D}} P_{D,i} = f(\overline{\omega}), \tag{9b}$$

$$\underline{P}_{G,i} \leq P_{G,i} \leq \overline{P}_{G,i}, \quad i \in \mathcal{G}, \tag{9c}$$

$$\underline{P}_{D,i} \leq P_{D,i} \leq \overline{P}_{D,i} \quad i \in \mathcal{D} \tag{9d}$$

where $C_{G,i}(P_{G,i}), -U_{D,i}(P_{D,i}) \in C^2$ and strictly convex. These functions that appear in the objective function denote the generation cost functions and users' disutility functions, respectively. Observe that, the objective function (9a) reflects the negative of the Global Social Welfare. The constraint given by (9b) is the power balance constraint where the function f maps the average frequency deviations to the grid's real-time power mismatch between supply and demand that has to be eliminated. Suitable choices for the function f, are the negative of ACE or the integral of that. We note that, f depends only on the average frequency as here we only consider a single-area setting with no tie-lines. In general, any methodology that seeks to solve the OFC problem effectively has to guarantee that the generation and consumption power increments converge to an equilibrium (P^*_G, P^*_D) which is also the optimal solution of the OFC problem. In the sequel, we present a distributed Newton-like OFC algorithm that can realize fast convergence to the optimal point (P^*_G, P^*_D), of course within a sufficiently small neighborhood around the optimal solution. We note that exact characterization of the convergence rate of the proposed algorithm and comparison of that with the convergence rate of other methods is beyond the scope of this work but an interesting direction for future work.

B. Optimal Frequency Control Algorithm

We propose the following optimization algorithm for attaining solution to the OFC problem under the physical dynamics (1)-(4):

$$\dot{P}_D = (-\nabla^2 U_D)^{-1}\Big[\nabla U_D + G_4^\top \omega + q\cdot\mathbf{1} + q^\ell_D - q^u_D\Big] \tag{10}$$

$$\dot{P}_G = (\nabla^2 \tilde{C}_G)^{-1}\Big[-\nabla\tilde{C}_G - R^\dagger P_M - q\cdot\mathbf{1} + q^\ell_G - q^u_G\Big] \tag{11}$$

$$\dot{q} = \Big(\sum_{i\in\mathcal{G}} P_{G,i} - \sum_{i\in\mathcal{D}} P_{D,i} - f(\overline{\omega})\Big) \tag{12}$$

$$\dot{q}^\ell_D = \Big[\underline{P}_D - P_D\Big]^+_{q^\ell_D}, \ \dot{q}^u_D = \Big[P_D - \overline{P}_D\Big]^+_{q^u_D} \tag{13}$$

$$\dot{q}^\ell_G = \Big[\underline{P}_G - P_G\Big]^+_{q^\ell_G}, \ \dot{q}^u_G = \Big[P_G - \overline{P}_G\Big]^+_{q^u_G} \tag{14}$$

where:

$$R^\dagger := \mathrm{diag}(R_1, R_2, \ldots, R_{|\mathcal{G}|}), \ G_4 := \begin{bmatrix} G_{4G} \\ G_{4L}\end{bmatrix} \tag{15}$$

$$G_4(i,j) := \begin{cases} 1, & \text{if DR unit } j \text{ lies at bus } i \\ 0, & \text{otherwise}\end{cases}$$

and $\nabla\tilde{C}_G(P_G) := \nabla C_G(P_G) - R^\dagger P_G$ where $\tilde{C}_G$ is still strictly convex. Further, $[g]^+_\mu : \mathbb{R}^n \mapsto v$ is a vector projection operator whose output vector v is given by:

$$v_i = \begin{cases} g_i, & \mu_i > 0 \\ \max\{0, g_i\}, & \mu_i = 0,\end{cases} \tag{16}$$

When the above real-time OFC algorithm is adopted the primal and dual variables are guaranteed to converge to the saddle point $(P^*_G, P^*_D, q^*, q^{\ell*}_D, q^{u*}_D, q^{\ell*}_G, q^{u*}_G)$ of the Lagrangian $\mathcal{L}(\cdot)$ of the OFC problem where (P^*_G, P^*_D) corresponds to the optimal solution of the OFC problem. We note that, the algorithm is applicable only in the case where the Hessian matrices of the generation cost and utility functions are non-singular. The convergence of the OFC algorithm will be shown formally in the next section.

IV. Stability Analysis using Passivity Theory

In this section, our aim is twofold; firstly, to prove that the overall system composed of the coupled physical network dynamics (1) - (4) and OFC optimization algorithm dynamics (10)-(14) is passive and secondly, by leveraging this passivity property to establish global asymptotic stability of the equilibrium of the overall system. Before stating our main results, we review some background from Passivity theory to lay the necessary foundation.

A. Passivity Theory

We depart from the definition of a passive system [10].

Definition 1 (Passive System, [10]). *Consider the system:*

$$\dot{x} = f(x,u), \; y = h(x,u) \tag{17}$$

where f is locally Lipschitz, h is continuous, $f(0,0) = 0$ and $h(0,0) = 0$. This system is passive if there exists a continuous differentiable positive semidefinite function $S(x)$ called the storage function such that it holds:

$$u^T y \geq \dot{S} = \frac{\partial S}{\partial x} f(x,u), \;\; \forall (x,u) \tag{18}$$

Therefore, proving that a given system is passive heavily relies on our ability to successfully choose a proper input-output combination jointly with a positive semidefinite storage function that satisfy (18). Depending on the complexity of the system, this task can be extremely hard or even impossible. On the bright side, there exist various theoretical results that establish passivity of certain classes of dynamical systems. A particular result that is applicable to the physical system given by (1)-(4) is the following.

Proposition 1 ([11]). *Consider the system described by:*

$$R\ddot{x} + Q\dot{x} + Px = u, \; y = r\dot{x} \tag{19}$$

where $u \in R^n$ is the input, $y \in R^n$ the output and $x \in R^n$ the state-variable. Let R, Q and $P \in R^{n\times n}$ be positive definite matrices. Then, this system is passive.

Proposition 1 and Definition 1 stem from Passivity theory and form the necessary background. With this, we are ready to derive our results. We begin with the system composed of the generator and load buses dynamics.

B. Passivity of the System of Generator and Load Buses

Before proceeding to show passivity of the system of generator and load buses, we define the following vectors:

$$\delta := \begin{bmatrix}\delta_G^\top & \delta_L^\top\end{bmatrix}^\top \tag{20}$$
$$\delta_G := \begin{bmatrix}\delta_{G,1} & \delta_{G,2} & \cdots & \delta_{G,|\mathcal{N}_G|}\end{bmatrix}^\top \tag{21}$$
$$\delta_L := \begin{bmatrix}\delta_{L,1} & \delta_{L,2} & \cdots & \delta_{L,|\mathcal{N}_L|}\end{bmatrix}^\top \tag{22}$$
$$\omega := \begin{bmatrix}\omega_G^\top & \omega_L^\top\end{bmatrix}^\top := \begin{bmatrix}\dot{\delta}_G^\top & \dot{\delta}_L^\top\end{bmatrix}^\top \tag{23}$$
$$P_M := \begin{bmatrix}P_{M,1} & P_{M,2} & \cdots & P_{M,|\mathcal{G}|}\end{bmatrix}^\top \tag{24}$$
$$P_{St} := \begin{bmatrix}P_{St,1} & P_{St,2} & \cdots & P_{St,|\mathcal{G}|}\end{bmatrix}^\top \tag{25}$$
$$P_D := \begin{bmatrix}P_{D,1} & P_{D,2} & \cdots & P_{D,|\mathcal{D}|}\end{bmatrix}^\top \tag{26}$$
$$P_L := \begin{bmatrix}P_{LG}^\top & P_{LL}^\top\end{bmatrix}^\top \tag{27}$$
$$P_{LG} := \begin{bmatrix}P_{L,1} & P_{L,2} & \cdots & P_{L,|\mathcal{N}_G|}\end{bmatrix}^\top \tag{28}$$
$$P_{LL} := \begin{bmatrix}P_{L,|\mathcal{N}_G|+1} & P_{L,|\mathcal{N}_G|+2} & \cdots & P_{L,|\mathcal{N}_G|+|\mathcal{N}_L|}\end{bmatrix}^\top \tag{29}$$
$$M := \mathrm{diag}(M_G, \underbrace{0, 0, \cdots, 0}_{|\mathcal{N}_L|}) \tag{30}$$
$$M_G := \mathrm{diag}(M_1, M_2, \ldots, M_{|\mathcal{N}_G|}) \tag{31}$$
$$D := \mathrm{diag}(D_G, D_L) \tag{32}$$
$$D_G := \mathrm{diag}(D_1, D_2, \ldots, D_{|\mathcal{N}_G|}) \tag{33}$$
$$D_L := \mathrm{diag}(D_{|\mathcal{N}_G|+1}, D_{|\mathcal{N}_G|+2}, \ldots, D_{|\mathcal{N}_G|+|\mathcal{N}_L|}) \tag{34}$$
$$G_3 := \begin{bmatrix}G_3(i,j)\end{bmatrix} \text{ where:} \tag{35}$$
$$G_3(i,j) := \begin{cases} 1, & \text{if gen and storage } j \text{ lies at bus } i \\ 0, & \text{otherwise} \end{cases} \tag{36}$$
$$G_5 := \begin{bmatrix}G_5(i,j)\end{bmatrix} \text{ where:} \tag{37}$$
$$G_5(i,j) := \begin{cases} 1, & \text{if load } j \text{ lies at bus } i \\ 0, & \text{otherwise} \end{cases} \tag{38}$$
$$T := \{T_{ij}\} =: \begin{bmatrix}T_G \\ T_L\end{bmatrix} : \text{ graph Laplacian} \tag{39}$$

The physical network can be expressed in compact form as:

$$\Lambda_1 : \quad M\ddot{\delta} + D\dot{\delta} + T\delta = G_3(P_M + P_{St}) - G_4 P_D - G_5 P_L \tag{40}$$

Recall that Λ_1 refers to the physical network which consists of the coupled dynamics of the generator and load buses. We establish passivity of Λ_1 through the following proposition.

Proposition 2. *The System Λ_1 with input $u_1 = G_3(P_M + P_{St}) - G_4 P_D - G_5 P_L$ and output $y_1 = \omega$ is passive.*

Proof. Let the state-space vector of Λ_1 be defined as:

$$\mathcal{X}_1 := \begin{bmatrix}\delta_G^\top & \delta_L^\top & \dot{\delta}_G^\top\end{bmatrix}^\top \tag{41}$$

The power input flow into Λ_1 can be computed as:

$$\begin{aligned} u_1^\top y_1 = & \left(\begin{bmatrix}M_G & O \\ O & O\end{bmatrix} \begin{bmatrix}\ddot{\delta}_G \\ \ddot{\delta}_L\end{bmatrix} + \begin{bmatrix}D_G & O \\ O & D_L\end{bmatrix} \begin{bmatrix}\dot{\delta}_G \\ \dot{\delta}_L\end{bmatrix} \right. \\ & \left. +T \begin{bmatrix}\delta_G \\ \delta_L\end{bmatrix} \right)^\top \begin{bmatrix}\dot{\delta}_G \\ \dot{\delta}_L\end{bmatrix} \\ = & \ddot{\delta}_G^\top M_G \dot{\delta}_G + \dot{\delta}_G^\top D_G \dot{\delta}_G + \dot{\delta}_L^\top D_L \dot{\delta}_L + \begin{bmatrix}\delta_G \\ \delta_L\end{bmatrix}^\top T \begin{bmatrix}\dot{\delta}_G \\ \dot{\delta}_L\end{bmatrix} \end{aligned} \tag{42}$$

We construct a storage function $S_1(\mathcal{X}_1)$ for Λ_1 as:

$$\begin{aligned} S_1(\mathcal{X}_1) := & \frac{1}{2} \mathcal{X}_1^\top \begin{bmatrix}T & O \\ O & M_G\end{bmatrix} \mathcal{X}_1 \\ = & \frac{1}{2} \dot{\delta}_G^\top M_G \dot{\delta}_G + \frac{1}{2} \begin{bmatrix}\delta_G \\ \delta_L\end{bmatrix}^\top T \begin{bmatrix}\delta_G \\ \delta_L\end{bmatrix}, \end{aligned} \tag{43}$$

and compute its time derivative as:

$$\dot{S}_1(\mathcal{X}_1) = \ddot{\delta}_G^\top M_G \dot{\delta}_G + \begin{bmatrix}\dot{\delta}_G \\ \dot{\delta}_L\end{bmatrix}^\top T \begin{bmatrix}\delta_G \\ \delta_L\end{bmatrix}. \tag{44}$$

By combining (42) and (44), we finally obtain the relation:

$$u_1^\top y_1 = \dot{S}_1(\mathcal{X}_1) + \omega^\top \mathrm{diag}(D_G, D_L)\omega \geq \dot{S}_1(\mathcal{X}_1). \tag{45}$$

From (45), we conclude that the system Λ_1 is passive. With that, we complete the proof. ∎

Notice that the physical system is made up of two subsystems, the subsystem of generator and load buses dynamics and the subsystem of governor-prime mover dynamics. Proposition 2 establishes passivity of the former subsystem.

We move on to establish passivity of the latter.

C. Passivity of Governor- Prime Mover Model

The combined governor and prime mover dynamics can be written succinctly in matrix form as:

$$\Lambda_2: \quad \dot{P}_M = \tau^{-1}(P_G - P_M - R^*\omega) \tag{46}$$
$$y := P_M^\dagger := R^\dagger P_M \tag{47}$$
$$P_G := \begin{bmatrix} P_{G,1} & P_{G,2} & \cdots & P_{G,|\mathcal{G}|} \end{bmatrix}^\top \tag{48}$$
$$\tau := \mathrm{diag}(\tau_1, \tau_2, \ldots, \tau_{|\mathcal{G}|}) \tag{49}$$
$$R^* := (R^\dagger)^{-1} G_3^\top \tag{50}$$

Note that, the matrix $G_3^\top$ maps the full frequency vector ω to the vector ω_G with its elements permuted. We establish passivity of Λ_2 through the following proposition.

Proposition 3. *The System Λ_2 with input $u_2 = (P_G - R^*\omega)$ and output $y_2 = P_M^\dagger$ is passive.*

Proof. The power flow into Λ_2 can be computed as:

$$\begin{aligned} u_2^\top y_2 &= (P_G - R^*\omega)^\top R^\dagger P_M \\ &= P_G^\top R^\dagger P_M - \omega^\top {R^*}^\top R^\dagger P_M \end{aligned} \tag{51}$$

Let the state-vector $\mathcal{X}_2$ and the storage function $S_2(\mathcal{X}_2)$ be:

$$\mathcal{X}_2 := P_M, \;\; S_2(\mathcal{X}_2) := \frac{1}{2}\mathcal{X}_2^\top R^\dagger \tau \mathcal{X}_2, \tag{52}$$

Time differentiation of S_2 gives:

$$\begin{aligned} \dot{S}_2(\mathcal{X}_2) &= \frac{1}{2}\dot{\mathcal{X}}_2^\top R^\dagger \tau \mathcal{X}_2 + \frac{1}{2}\mathcal{X}_2^\top R^\dagger \tau \dot{\mathcal{X}}_2 \\ &= [\tau^{-1}(P_G - P_M - R^*\omega)]^\top R^\dagger \tau P_M \\ &= P_G^\top R^\dagger P_M - P_M^\top R^\dagger P_M - \omega^\top {R^*}^\top R^\dagger P_M \end{aligned} \tag{53}$$

By combining (51), (53) and $P_M^\top R^\dagger P_M > 0$ we obtain:

$$u_2^\top y_2 = \dot{S}_2(\mathcal{X}_2) + P_M^\top R^\dagger P_M \geq \dot{S}_2(\mathcal{X}_2) \tag{54}$$

Therefore, we deduce that Λ_2 is output strictly passive. ∎

We continue to show passivity of the overall physical network. Prior to that, we state the transfer function system representations of Λ_1 and Λ_2 as:

$$\Lambda_1: \omega = \dot{\delta} = F_1(s)(G_3(P_M + P_{St}) - G_4 P_D - G_5 P_L) \tag{55}$$
$$\Lambda_2: P_M^\dagger = F_2(s)(P_G - R^*\omega) \tag{56}$$
$$F_1(s) := s(s^2 M + sD + T)^{-1} \tag{57}$$
$$F_2(s) := R^\dagger (sI + \tau^{-1})^{-1}\tau^{-1} = R^\dagger (s\tau + I)^{-1} \tag{58}$$

The utility of this description will become clear in the sequel.

D. Passivity of the Overall Physical Network

So far, we established that both the system of the generator and load buses and the system of governors are passive when considered decoupled. Next, we assemble these properties to establish passivity of the overall coupled system through the following lemma.

Lemma 1. *The system constructed by the negative feedback connection of Λ_1 and Λ_2 is passive with input $v = \begin{bmatrix} v_1^\top & v_2^\top \end{bmatrix}^\top$ and output $y = \begin{bmatrix} y_1^\top & y_2^\top \end{bmatrix}^\top$ given precisely by:*

$$y_1 := \omega \tag{59}$$
$$u_1 := G_3(P_M + P_{St}) - G_4 P_D - G_5 P_L = {R^*}^\top y_2 + v_1 \tag{60}$$
$$v_1 := G_3 P_{St} - G_4 P_D - G_5 P_L, \;\; y_2 := R^\dagger P_M = P_M^\dagger \tag{61}$$
$$u_2 := P_G - R^*\omega = v_2 - R^* y_1, \;\; v_2 := P_G \tag{62}$$

Proof. The input flow into the overall system is:

$$\begin{aligned} &y^\top v = y_1^\top v_1 + y_2^\top v_2 = y_1^\top u_1 + y_2^\top u_2 \\ &= \dot{S}_1 + \dot{S}_2 + \omega^\top \mathrm{diag}(D_G, D_L)\omega + P_M^\top R^\dagger P_M \geq \dot{S}_1 + \dot{S}_2 \end{aligned} \tag{63}$$

The last inequality arises by combining Propositions 2 and 3. From that, we conclude that the overall physical system $\Lambda_1 - \Lambda_2$ is passive with storage function $S_1 + S_2$. ∎

Interestingly, the overall physical system can be viewed as the negative feedback connection of the systems Λ_1 and Λ_2.

E. Passivity of Optimal Frequency Control System

Naturally, the next step is to show passivity of the OFC system. This is carried out by exploiting the following result.

Proposition 4 ([12]). *Consider the iteration algorithm:*

$$\dot{v} = (-(\nabla^2 V(v))^{-1})(\nabla V(v) + R^\top p) \tag{64}$$
$$p = f(s)(-Rv + b) \tag{65}$$

which attains solution to the problem:

$$\underset{v}{\text{maximize}} \; V(v) \tag{66}$$
$$\text{subject to}: Rv - b = 0 \tag{67}$$
$$V(v) \in C^2 \text{and strictly concave}, \; R: \text{full row rank}$$

Let $f(s)$ be a positive real transfer function with a simple pole at the origin. Then, the systems (64) *and* (65) *are passive with input and output pairs $(R^\top \hat{p}, \hat{v})$ and $(-R\hat{v}, \hat{p})$, respectively, where $\hat{v} = v - v^*$, $\hat{p} = p - p^*$, and (v^*, p^*) is the equilibrium point of* (64) *and* (65)*. Moreover, the equilibrium (v^*, p^*) is globally asymptotically stable and corresponds to the optimal solution of* (66) - (67).

Armed with Proposition 4, we state the following result for the OFC system (10)-(14).

Theorem 1. *The OFC system described by:*

$$\Lambda_3: \dot{P}_D = (-\nabla^2 U_D)^{-1}\Big[\nabla U_D + u_3 + q \cdot \mathbf{1} + q_D^\ell - q_D^u\Big] \tag{68}$$
$$\Lambda_4: \dot{P}_G = (\nabla^2 \tilde{C}_G)^{-1}\Big[- \nabla \tilde{C}_G + u_4 - q \cdot \mathbf{1} + q_G^\ell - q_G^u\Big] \tag{69}$$

$$\Lambda_5 : \dot{q} = -\Big(\sum_{i \in \mathcal{D}} P_{D,i} - \sum_{i \in \mathcal{G}} P_{G,i} + f(\overline{\omega}) \Big) \tag{70}$$

$$\Lambda_6 : \dot{q}_D^l = \Big[\underline{P}_D - P_D \Big]^+_{q_D^\ell}, \ \Lambda_7 : \dot{q}_D^u = \Big[P_D - \overline{P}_D \Big]^+_{q_D^u} \tag{71}$$

$$\Lambda_8 : \dot{q}_G^l = \Big[\underline{P}_G - P_G \Big]^+_{q_G^\ell}, \ \Lambda_9 : \dot{q}_G^u = \Big[P_G - \overline{P}_G \Big]^+_{q_G^u} \tag{72}$$

is incrementally passive where $\begin{bmatrix} u_3^\top & u_4^\top & -f(\overline{\omega}) \end{bmatrix}^\top$ *is the input and* $\begin{bmatrix} P_D^\top & P_G^\top & q \end{bmatrix}^\top$ *is the output.*

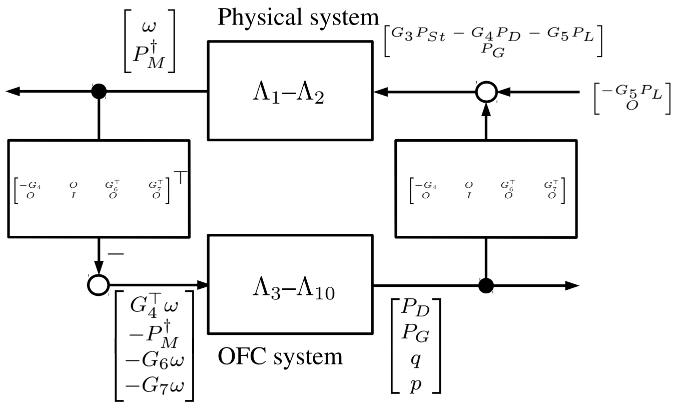

Fig. 1: Overall system as a negative feedback connection of Λ_1–Λ_2 (Physical system) and Λ_3–Λ_{10} (OFC system).

Realize that our proposed OFC system (10)-(14) can arise from the system in Theorem 1 in the special case where $u_3 = G_4^\top \omega$ and $u_4 = -R^\dagger P_M$. Therefore, a corollary of Theorem 1 is that the system (10)-(14) is passive. To carry on, we define:

$$f(\overline{\omega}) := G_6 \omega, \ \ G_6 := \frac{1}{|\mathcal{N}_G|} \begin{bmatrix} \mathbf{1}^\top & \mathbf{0}^\top \end{bmatrix} \tag{73}$$

$$\Lambda_{10} : \dot{p} = -\omega_G = -G_7 \omega, \ G_7 = \begin{bmatrix} I & O \end{bmatrix}. \tag{74}$$

Below, we state our final theorem that establishes asymptotic stability of the overall system.

Theorem 2. *The overall system* $\Lambda_1 - \Lambda_{10}$ *composed of the physical network* $\Lambda_1 - \Lambda_2$ *and the OFC system* $\Lambda_3 - \Lambda_{10}$ *with:*

$$u_3 = G_4^\top \omega, \ u_4 = -P_M^\dagger, \ f(\overline{\omega}) = G_6 \omega \tag{75}$$

$$G_3 P_{St} = G_6^\top q + G_7^\top p \tag{76}$$

$$v_1 = G_3 P_{St} - G_4 P_D - G_5 P_L, \ v_2 = P_G, \tag{77}$$

has a asymptotically stable equilibrium that satisfies:

$$\omega_G^* = O \tag{78}$$

$$- T\delta^* + G_3 P_M^* - G_4 {P_D}^* - G_5 P_L + G_6^\top q^* = O \tag{79}$$

$$P_M^* = P_G^* \tag{80}$$

$$\nabla U_D({P_D}^*) + \mathbf{1} q^* + q_D^{\ell *} - q_D^{u *} = O \tag{81}$$

$$- \nabla C_G({P_G}^*) - \mathbf{1} q^* + q_G^{\ell *} - q_G^{u *} = O \tag{82}$$

$$\mathbf{1}^\top {P_D}^* - \mathbf{1}^\top {P_G}^* + G_6 \omega^* = 0. \tag{83}$$

Moreover, the equilibrium $(P_D^*, P_G^*, q^*, q_\bullet^{\bullet *}, p^*)$ *corresponds to the optimal solution of the OFC problem.*

In summary, we have proposed an OFC algorithm that can result in optimal frequency regulation. We have proved that the physical and the OFC subsystems are passive when viewed decoupled. Lastly, by combining these properties we have showed that these two subsystems give rise to a passive overall system that has an asymptotically stable equilibrium.

V. Concluding Remarks

Power systems with high penetration of RERs experience large frequency fluctuations. That calls for rethinking the current practice of secondary frequency control (SFC) with aim to improve its economic efficiency. With this in mind, in this paper we propose an OFC system that enables generators and DR units to carry out optimal frequency regulation with minimum cost. Our control methodology realizes high economic efficiency by allowing for online convergence of the SFC set-points to their optimal values. We establish guarantees for the dynamic performance of a power network that adopts the proposed scheme through a scalable and computationally efficient methodology based on passivity theory.

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