Performance Competition in Cooperative Capturing by Multi-agent Systems

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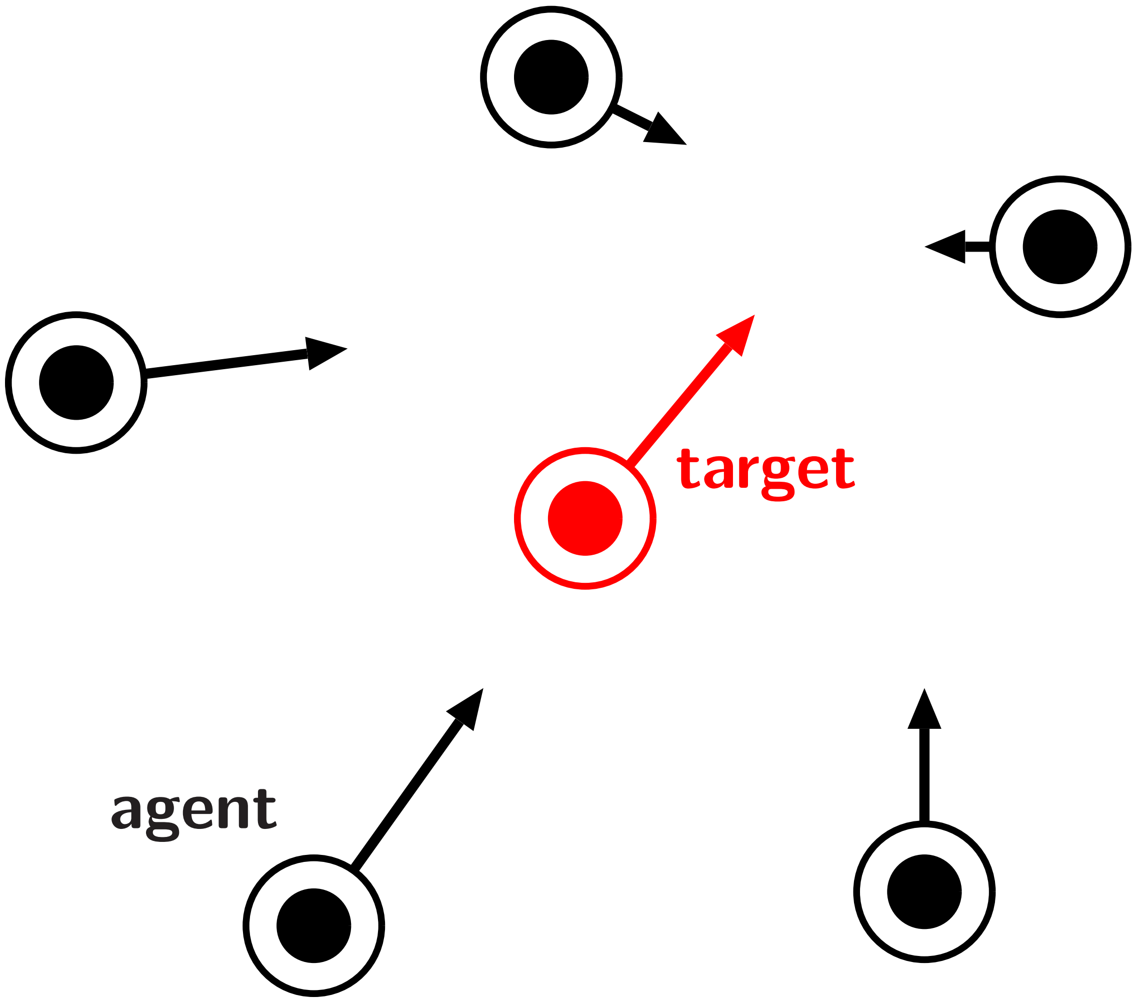
Introduction

strategy for capturing vs escape



control theoretic interpretation of the success of capturing or escape with respect to their performances?

History

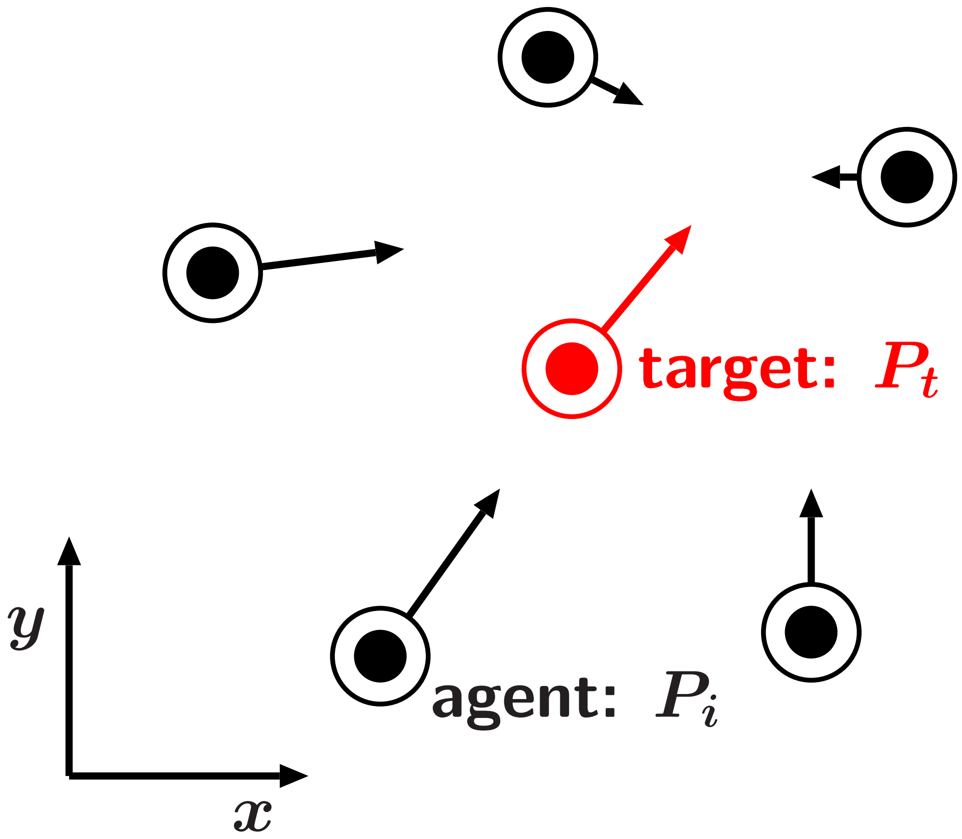
- formation control: flocking of robots, vehicles, ... or biological systems Reynolds et al. (87), Desai et al. (01), Egerstedt & Hu (01), Leonard & Fiorello (01), Olfati-Saber & Murray (02), Tanner et al. (03), Marshall et al. (04)
- cooperative cyclic pursuit: Sinha & Ghose (06), Marshall et al. (04), Marshall et al. (06), Kobayashi et al. (06), Sepulchre et al. (06)

Kim & Sugie (07): simple strategy

Hara et al. (07): general dynamics

"the target does not move or moves regardless of the agents"

Formulation



P_1, P_2, . . . , P_n: agents

P_t: target

x–y plane

coordinates: $p_i(t) = \begin{bmatrix} x_i(t) \\ y_i(t) \end{bmatrix} \in \mathbb{R}^2, \; p_t(t) = \begin{bmatrix} x_t(t) \\ y_t(t) \end{bmatrix} \in \mathbb{R}^2$

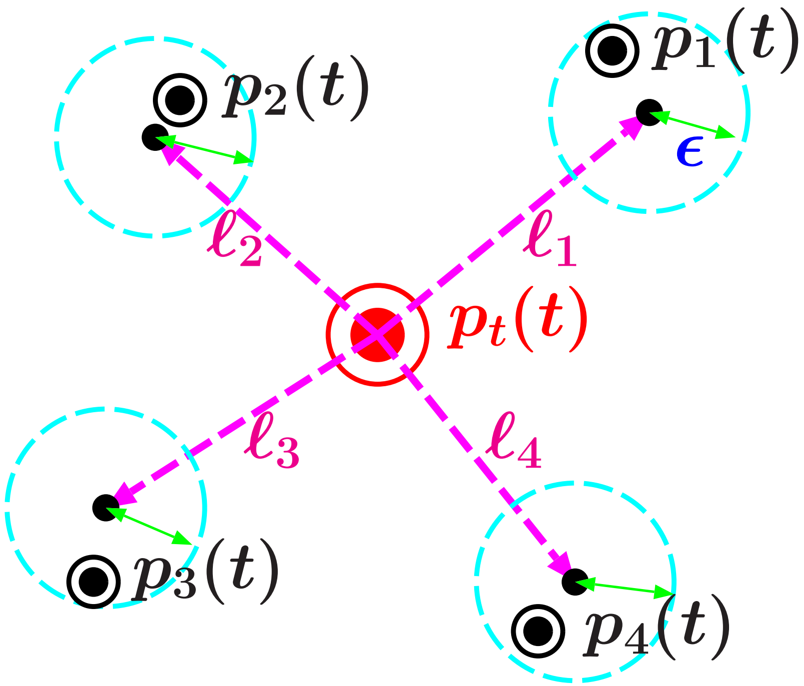
dynamics: $\dot{p}_i = f_i(p_i, u_i), \; \dot{p}_t = f_t(p_t, u_t)$

Definition of Capturing

When for any given $\epsilon > 0$, there exist a constant $\delta > 0$ and for any initial conditions $p_i(0)$, $p_t(0)$ such as $\|p_i(0) - \{p_t(0) + l_i\}\| < \delta$,

$$\|p_i(t) - \{p_t(t) + l_i\}\| < \epsilon, \quad \forall t$$

is satisfied, then we call the **capturing is attained**.



Cooperative Capturing: case of simple dynamics

$$\dot{p}_i = u_i$$
$$\dot{p}_t = u_t$$

control law of the agents:

$$u_i = -k_i\{p_i - (p_t + l_i)\}$$
$$k_i := \begin{bmatrix} k_{x,i} & 0 \\ 0 & k_{y,i} \end{bmatrix},\ k_{x,i} > 0,\ k_{y,i} > 0$$

k_i: performance index of the agents

Escape strategy of the target:

$$u_t = \sum_{i=1}^{n} \alpha_i k_t (p_t - p_i),\ \ k_t := \begin{bmatrix} k_{x,t} & 0 \\ 0 & k_{y,t} \end{bmatrix},\ \alpha_i := \begin{bmatrix} \alpha_{x,i} & 0 \\ 0 & \alpha_{y,i} \end{bmatrix},$$

$$k_{x,t},\ k_{y,t},\ \alpha_{x,i},\ \alpha_{y,i} > 0$$

$$\alpha_{x,1} + \alpha_{x,2} + \cdots + \alpha_{x,n} = 1,\ \alpha_{y,1} + \alpha_{y,2} + \cdots + \alpha_{y,n} = 1$$

k_t: performance index of the target

α_i: weights on the agents

Condition for Capturing

Theorem (simple dynamics case) (KT et al. (2011))

A necessary and sufficient condition for the capturing is

$$k_t < \left(\sum_{i=1}^{n} \frac{\alpha_i}{k_i} \right)^{-1} . \quad (*)$$

Performance as a group

RHS of $(*)$: weighted harmonic mean of k_i

$\Rightarrow$ the performance of the group of the agents

Strategy for the agents

$$\left(\sum_{i=1}^{n} \frac{\alpha_i}{k_i}\right)^{-1} \le \sum_{i=1}^{n} \alpha_i k_i$$

The upper bound is attained at $k_1 = k_2 = \cdots = k_n =: k$.

$\Rightarrow$ **"homogeneous" group** is best for the agents

$(*) \Rightarrow \qquad k_t < k$

Strategy for the target

tuning the weights α_i for the target

when $k_1 > k_2 > \cdots > k_n$

$\Rightarrow \alpha_1 < \alpha_2 < \cdots < \alpha_n$ is a preferable setting

"Strong escape from weak agents is a preferable strategy for the target"