Adaptive Consensus of Discrete-time Heterogeneous Multi-agent Systems

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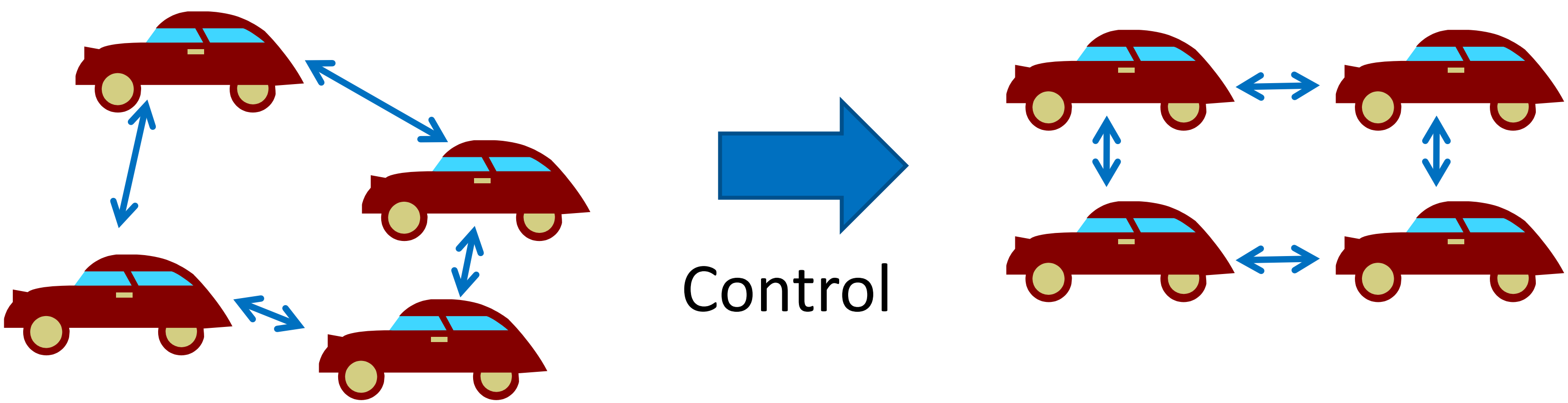
Multi-Agent Dynamical Systems (MADSs)

- Active research field [Fax & Murray 2004] [Olfati-Saber *et al.* 2007]
- Application for engineering
  - Cooperation control of robots
  - Vehicle formation
    - Unmanned Aerial Vehicles (UAVs)
    - Automated Highway Systems (AHSs)

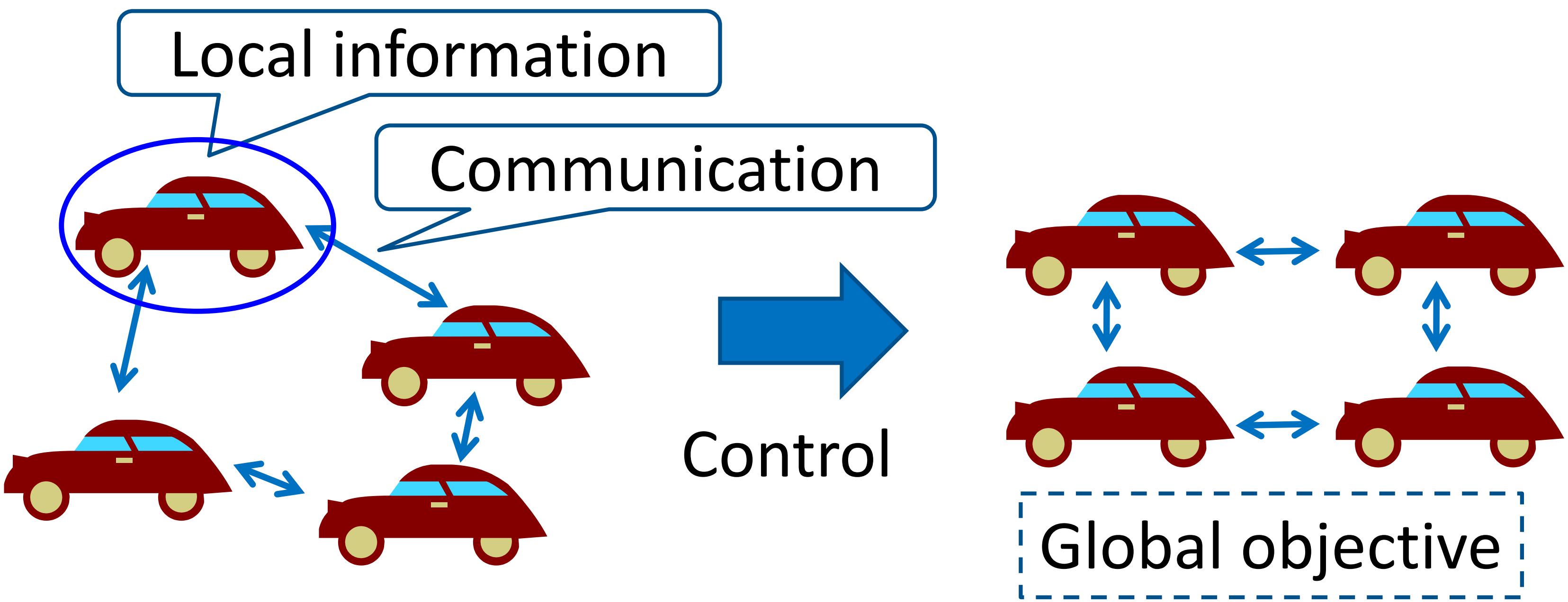


J.A. Fax and R.M. Murray, "*Information flow and cooperative control of vehicle formations*", 2004.

R. Olfati- Saber *et al.* "*Consensus and cooperation in networked multi-agent systems*", 2007.

Multi-Agent Dynamical Systems (MADSs)

- Agent = Dynamical system (with controller)
  - communicates with neighbor agents.
  - autonomously acts on local information.
- The system as a whole achieves some objective.



Consensus problem of MADSs

- Consensus problem
  - To reach an agreement on a *certain quantity*

In this presentation,

we consider *state consensus* problem.

$$\lim_{k\to\infty}(x_i(k) - x_j(k)) = 0, \quad \forall i, j$$

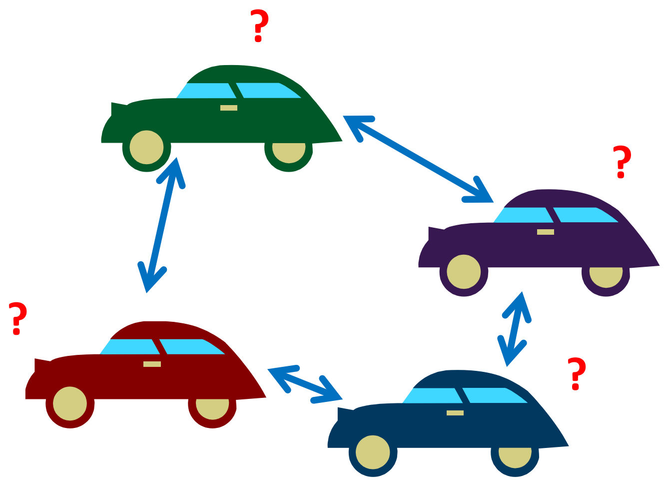
$x_i(k)$: state vector of i'th agent (at time step k)

Research objective

- *Homogeneous* MADSs have been mainly researched.
  [Fax & Murray 2004] [Olfati-Saber *et al.* 2007]

- Large number of the agents
  ➡ The dynamics of them are
  heterogeneous & *uncertain.*

  [Wieland *et al.* 2011] :heterogeneous
  [Kim *et al.* 2011] :heterogeneous & uncertain



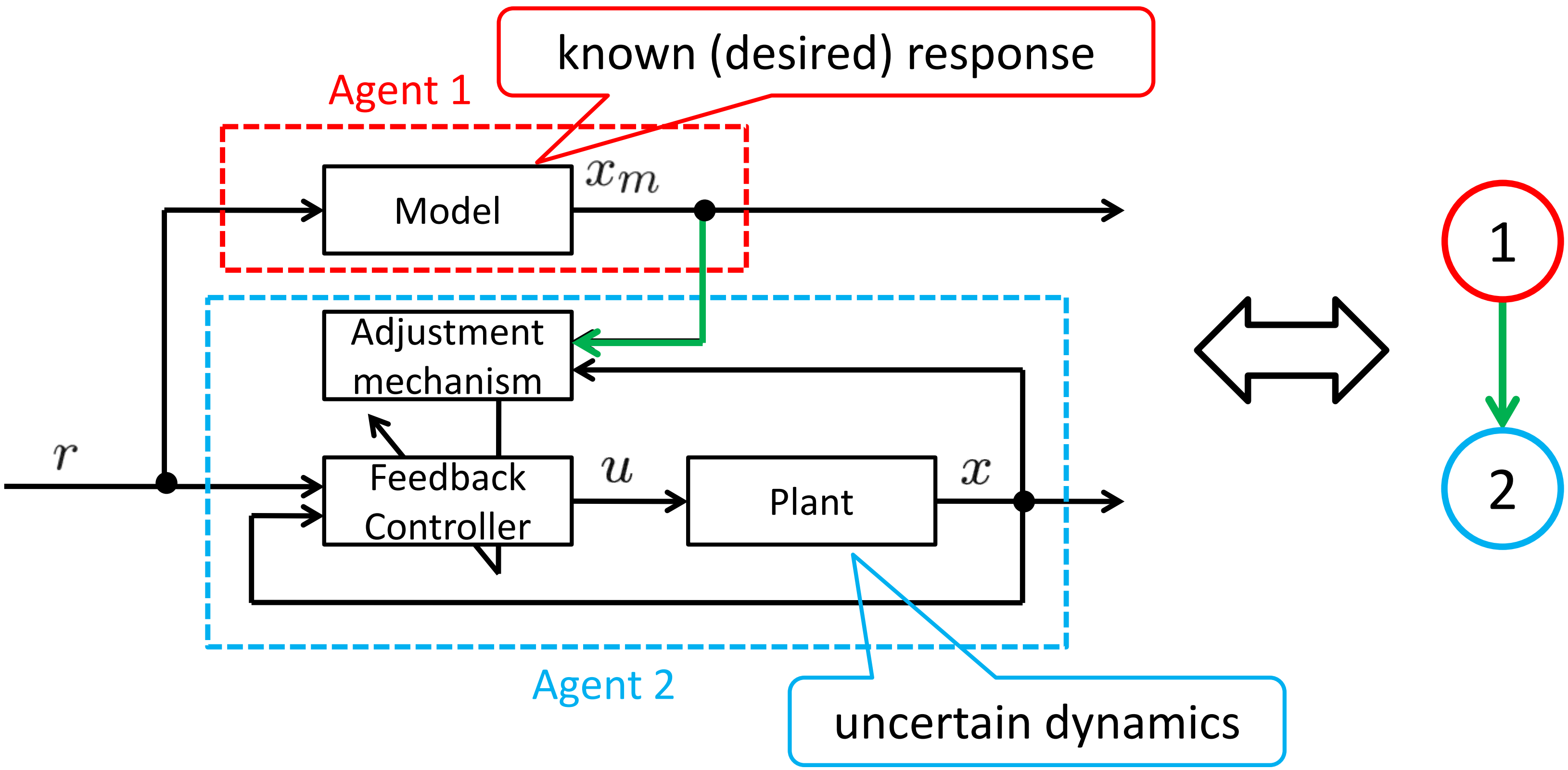
> state consensus control of
> uncertain heterogeneous MADSs

P. Wieland *et al*. "*An internal model principle is necessary and sufficient for linear output synchronization,*" 2011.
H. Kim *el al*. "*Output consensus of heterogeneous uncertain linear multi-agent systems*", 2011.

Contents

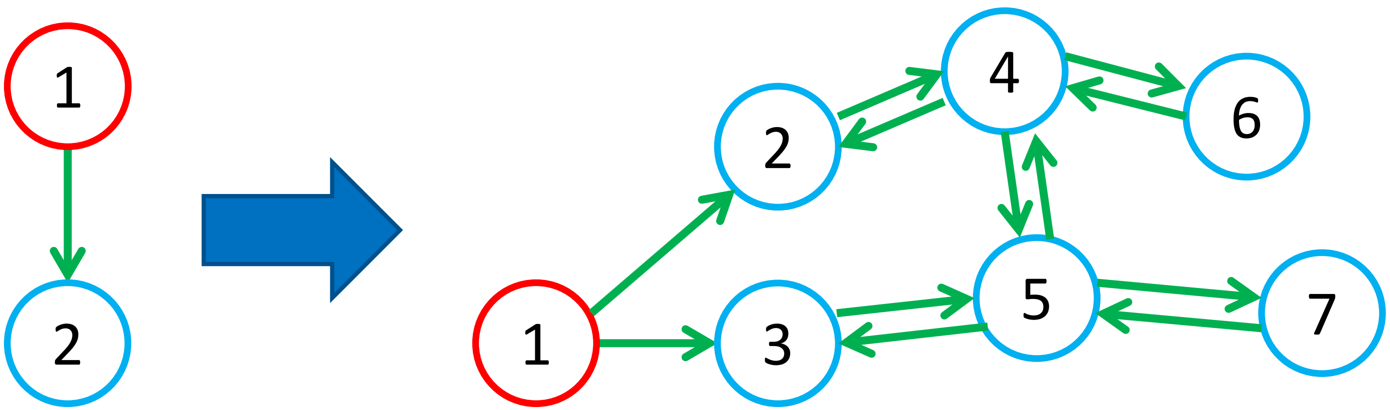
Adaptive control approach (1)

- A standard adaptive control can be regarded as a consensus problem of 2 agents.
- Extension to more general communication topology.

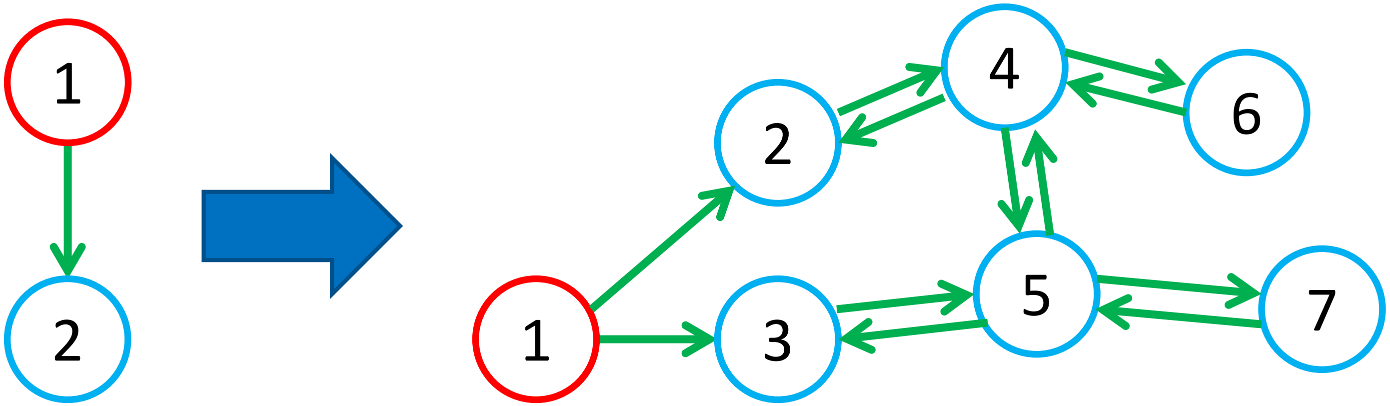


Adaptive control approach (2)

- A standard adaptive control can be regarded as a consensus problem of 2 agents.
- Extension to more general communication topology.



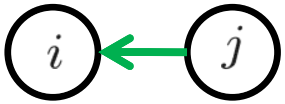
Adaptive control approach (3)



- Previous study (Adaptive control approach)
  - Kaizuka & Tsumura, 2010.
    - *continuous-time* systems
- **Corresponding result for *discrete-time* systems** is *not* straightforward.

Contents

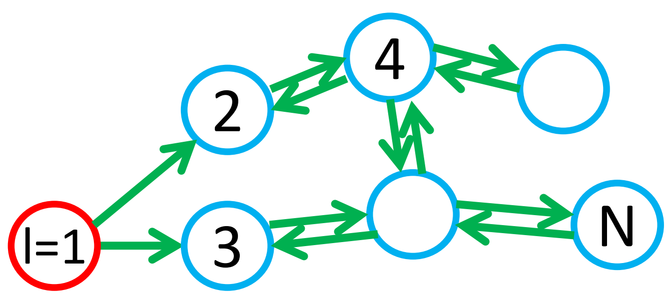
Communication topology

- Edge 

  ⇔ Agent i receives information of Agent j.
- A special agent $l = 1$ is called leader. (⇔known model)
- The other agent $(2, \cdots, N)$ are called followers.(⇔uncertain plants)

Assumption about Communication topology

- There are no edges *to* the leader.
- Edges between followers are *bidirectional*.
- There are directed spanning trees.



Heterogeneous dynamics of agents

Agent Dynamics state: $\mathbb{R}^n$ input: $\mathbb{R}^m$

Leader : $x_l(k+1) = A_l x_l(k) + Br(k)$

Follower : $x_i(k+1) = A_i x_i(k) + Bu_i(k) \quad (i = 2, \ldots, N)$

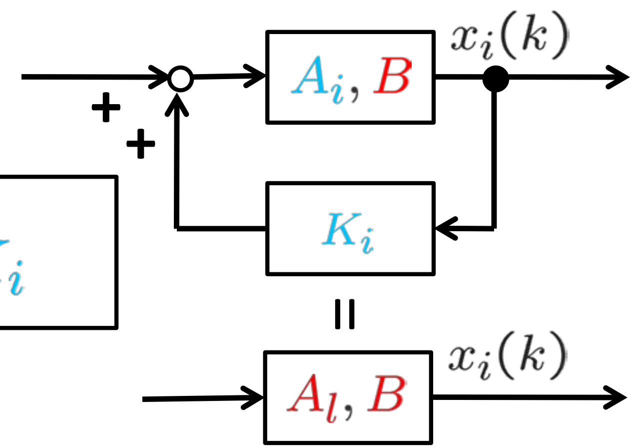
A_l :known, asymptotically stable

B :known, full column rank

A_i :unknown, heterogeneous, (possibly unstable)

$r(k)$:reference input, bounded

$u_i(k)$:control input to Agent i



Assumption (Matching condition)

$$\forall i \in \{2, \ldots, N\}, \ \exists K_i, \quad A_l = A_i + BK_i$$

K_i :unknown, heterogeneous

'the true value of gain'

State consensus problem

Agent Dynamics state: $\mathbb{R}^n$ input: $\mathbb{R}^m$

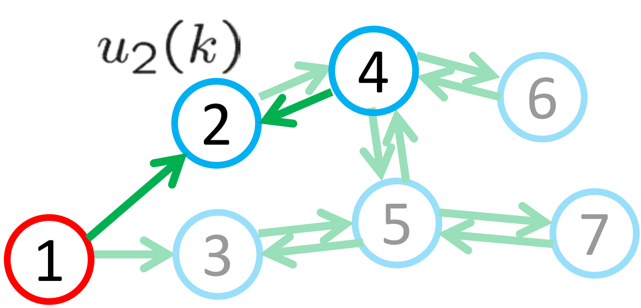
Leader : $x_l(k+1) = A_l x_l(k) + Br(k)$

Follower : $x_i(k+1) = A_i x_i(k) + Bu_i(k) \quad (i = 2, \ldots, N)$

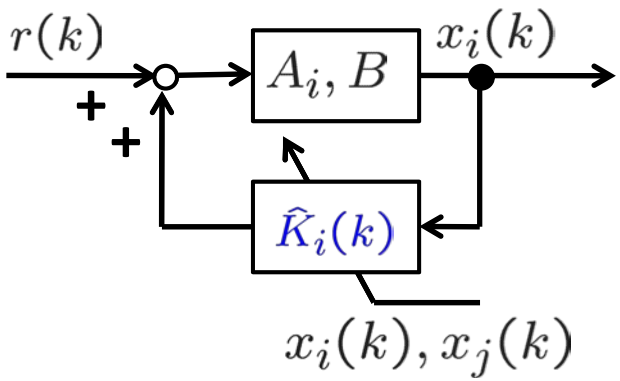
- Design control inputs $u_i(k)$.
  - Achieve state consensus.

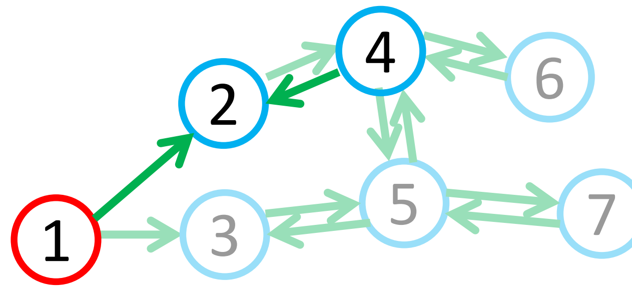
    $\lim_{k\to\infty}(x_i(k) - x_j(k)) = 0, \quad \forall i, j$

  - Use only the states of the *neighbors* and *its own*.



Adaptive Controller of Follower





Controller of Follower i

$\hat{K}_i(k)$:estimation (at time k) of K_i. ($A_l = A_i + BK_i$)

Feedback control law:

$$\boxed{u_i(k) = \hat{K}_i(k)x_i(k) + r(k)}$$

Gain update law:

$$\boxed{\hat{K}_i(k+1) = \hat{K}_i(k) - \sum_{j\in\mathcal{N}_i} \frac{Q_i B^{\dagger}(e_{ij}(k+1) - A_l e_{ij}(k))x_i^{\top}(k)}{c + x_i^{\top}(k)x_i(k) + x_j^{\top}(k)x_j(k)}}$$

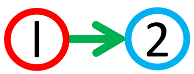
$e_{ij}(k) := x_i(k) - x_j(k)$

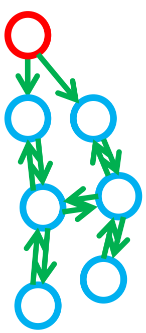
$\mathcal{N}_i$:Neighbors of agent i

$c > 0$, Q_i:positive definite s.t. $|\mathcal{N}_i|Q_i < 2I_m$

Gain update law

	Continuous-Time	Discrete-Time
standard	$\dot{\hat{K}}_2 = -B^\top P e_{2l} x_2^\top$	$\hat{K}_2(k+1) = \hat{K}_2(k) - \frac{Q_2 B^\dagger (e_{2l}(k+1) - A_l e_{2l}(k))}{c + x_2^\top(k) x_2(k)} x_2^\top(k)$ [Hayakawa *et al*. 2004]
		$\hat{K}_2(k+1) = \hat{K}_2(k) - \frac{Q_2 B^\dagger (e_{2l}(k+1) - A_l e_{2l}(k))}{c + x_l^\top(k) x_l(k) + x_2^\top(k) x_2(k)} x_2^\top(k)$
multi-agent	summation	summation
	$\dot{\hat{K}}_i = -B^\top P \left\{ \sum_{j \in \mathcal{N}_i} e_{ij} \right\} x_i^\top$ [Kaizuka & Tsumura 2010]	$\hat{K}_i(k+1) = \hat{K}_i(k) - \left\{ \sum_{j \in \mathcal{N}_i} \frac{Q_i B^\dagger (e_{ij}(k+1) - A_l e_{ij}(k))}{c + x_i^\top(k) x_i(k) + x_j^\top(k) x_j(k)} \right\} x_i^\top(k)$





Theorem

The controlled MADSs satisfies following.

- $\lim_{k\to\infty}(x_i(k) - x_j(k)) = 0, \quad \forall i, j$ (state consensus)
- $\sum_{i=2}^{N} \mathrm{tr}[\tilde{K}_i^\top Q_i^{-1} \tilde{K}_i]$ is non-increasing.

  ($\tilde{K}_i(k) := \hat{K}_i(k) - K_i$: Gain error at agent i)

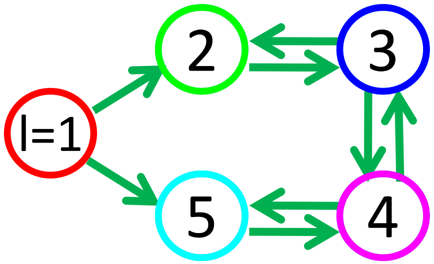
  $\Rightarrow$ The gain update law is a globally stable estimator.
- The system of {state errors & gain errors} is globally stable.

  (Details are given in the proceedings.)

- Introduction
- Adaptive control approach
- Framework
- Numerical example
- Conclusion

Numerical example

Communication topology



Agent Dynamics

Leader : $x_l(k+1) = A_l x_l(k) + Br(k)$

Follower : $x_i(k+1) = A_i x_i(k) + Bu_i(k)$

$$A_i = \begin{bmatrix} 0 & 1 \\ -a_{2,i} & -a_{1,i} \end{bmatrix}, B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

Poles

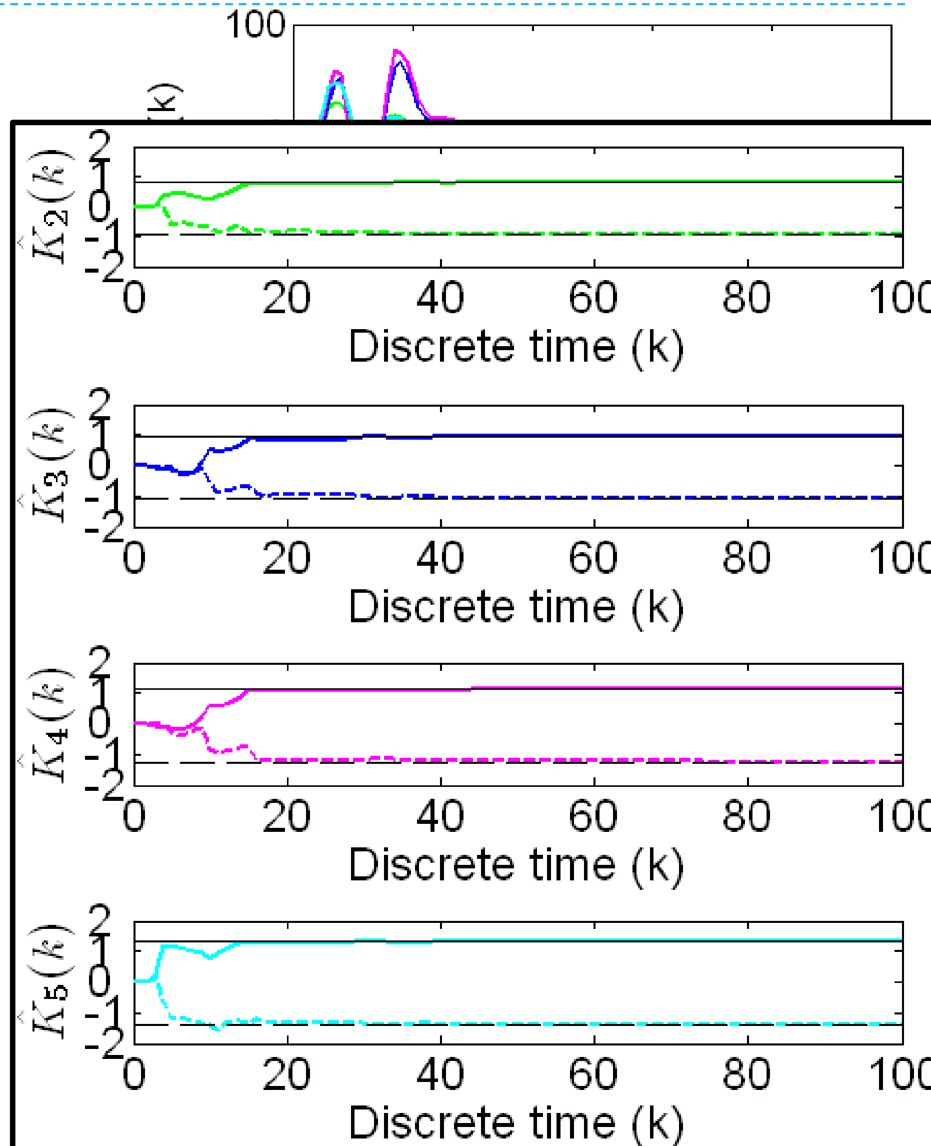
A_l $(0.5, 0.7)$ (stable)

A_i $(1+0.1(i-1), 1+0.05(i-1))$

$i = 2, \ldots, 5$ (unstable)

$$r(k) = 3\sin(2\pi k/10) \qquad x_i(k) = \begin{bmatrix} -10 \\ -10 \end{bmatrix}$$

$$\hat{K}(0) = \begin{bmatrix} 0 & 0 \end{bmatrix}$$

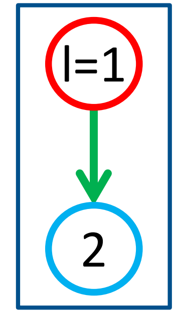


- Conclusions
  - We proposed a state consensus control framework for discrete-time uncertain heterogeneous MADSs.
  - We showed the adaptive control based framework is also valid for discrete-time systems.

- Future works
  - Output feedback case
  - The case that none of the agent dynamics is known.

Thank you for your attention!

Continuous-time vs. discrete-time



Continuous-time case

Lyapunov function: $V = e_{2l}^\top P e_{2l} + \mathrm{Tr}[\tilde{K}_2^\top \tilde{K}_2]$

$$\dot{V} = -e_{2l}^\top R e_{2l} + \cancel{2e_{2l}^\top P B \tilde{K}_2 x_2} - \cancel{2e_{2l}^\top P B \tilde{K}_2 x_2}$$

Linear!

Update law: $\dot{\hat{K}}_2 = -B^\top P e_{2l} x_2^\top$

$$\Rightarrow \dot{V} = -e_{2l}^\top R e_{2l} \le 0$$

$$PA_l + A_l^\top P = -R$$

$$e_{2l} := x_2 - x_l$$

$$\dot{e}_{2l}(t) = A_l e_{2l}(t) + B\tilde{K}_2(t)x_2(t)$$

Discrete-time case

$$\Delta(e_{2l}^\top P e_{2l}) = e_{2l}^\top (A_l^\top P A_l - P)e_{2l} + 2e_{2l}^\top A_l^\top P B \tilde{K}_2 x_2 + x_2^\top \tilde{K}_2^\top B^\top P B \tilde{K}_2 x_2$$

(Positive definite) cross term!

Lyapunov function:

$$V = \ln\left(1 + e_{2l}^\top P e_{2l}\right) + a\,\mathrm{tr}[\tilde{K}_2^\top Q_2^{-1} \tilde{K}_2]$$

Update law:

$$\hat{K}_2(k+1) = \hat{K}_2(k) - \frac{Q_2 B^\dagger (e_{2l}(k+1) - A_l e_{2l}(k)) x_2^\top(k)}{c + x_l^\top(k) x_l(k) + x_2^\top(k) x_2(k)}$$

$$\Rightarrow \Delta V \le -\frac{e_{2l}^\top R e_{2l}}{1 + e_{2l}^\top P e_{2l}} \le 0$$

$$(1+\varepsilon)A_l^\top P A_l - P + R < 0$$

$$e_{2l}(k+1) = A_l e_{2l}(k) + B\tilde{K}_2(k)x_2(k)$$

$a > 0$: large enough. $0 < Q_2 < 2I_m$